14. In Problem 11.13, if the height of the plate is 1 m, calculate the following:
    (i) heat transfer rate to the plate,
    (ii) maximum velocity of condensate at the trailing edge, and
    (iii) also, draw the variation of δ with distance from top.
15. Dry, saturated steam at atmospheric pressure condenses on a horizontal tube of diameter = 3 cm and height L = 1 m; surface of the tube is maintained at 80°C. Estimate the heat transfer rate and the mass of steam condensed per hour. Assume laminar film condensation.
    (b) If the tube is vertical, what is the condensation rate?
    [Hint: Use Eq. 11.62a].
16. Dry, saturated steam at atmospheric pressure condenses on a vertical tube of diameter = 5 cm and length L = 1.5 m; surface of the tube is maintained at 60°C. Determine the heat transfer rate and the mass of steam condensed per hour.
17. A steam condenser consists of a square array of 625 horizontal tubes, each 6 mm in diameter. The tubes are exposed to exhaust steam arriving from the turbine at a pressure of 0.15 bar. If the tube surface temperature is maintained at a temperature of 25°C by circulating cold water through the tubes, determine the heat transfer coefficient and the rate at which the steam is condensed per unit length of tubes for the entire array. Assume laminar film condensation and that there are no condensable gases mixed with steam.
18. A steam condenser consists of an array of horizontal tubes, each 2.0 cm in diameter and 1.5 m long. The tubes are arranged in such a manner that each vertical tier has 10 tubes; tubes are exposed to saturated steam at 100°C. If the tube surface temperature is maintained at a temperature of 80°C, determine the total number of tubes required to get a condensation rate of 0.4 kg/s. Assume laminar film condensation and that there are no condensable gases mixed with steam.
19. Ammonia at 40°C is condensing inside a horizontal tube of 25 mm ID. Mass velocity of ammonia vapour at inlet is 10 kg/(m^2s). Surface of the tube is maintained at a constant temperature of 20°C by circulating cold water. Calculate the fraction of vapour that will condense if the tube is 0.5 m long.
20. In Problem 19, if ammonia is condensing on the outside surface of the tubes, what will be the heat transfer rate?

CHAPTER

12

Heat Exchangers

12.1 Introduction

'Heat exchanger' is one of the most commonly used process equipments in industry and research. Function of a heat exchanger is to transfer energy; this transfer of energy may occur to a single fluid (as in the case of a boiler where heat is transferred to water) or between two fluids that are at different temperatures (as in the case of an automobile radiator where heat is transferred from hot water to air). In some cases, there are more than two streams of fluid exchanging heat in a heat exchanger. Heat exchangers of several designs in a variety of sizes varying from 'miniature' to 'huge' (with heat transfer areas of the order of 5000 to 10,000 sq. metres) have been developed over the years.

Some typical examples of heat exchanger applications are:

(i) Thermal power plants (boilers, superheaters, steam condensers, etc.)
(ii) Refrigeration and air-conditioning (evaporators, condensers, coolers)
(iii) Automobile industry (radiators, all engine cooling and fuel cooling arrangements)
(iv) Chemical process industry (variety of heat exchangers between different types of fluids, in cumbustors and reactors)
(v) Cryogenic industry (condenser–reboilers used in distillation columns, evaporators to produce gas from cryogenic liquids, etc.)
(vi) Research ('regenerators' used in Stirling engines, special ceramic heat exchangers used in ultra-low temperature devices, superconducting magnet systems, etc.).

12.2 Types of Heat Exchangers

Heat exchangers may be classified in several ways:

(i) according to heat exchange process
(ii) according to relative direction of flow of hot and cold fluids
(iii) according to constructional features, compactness, etc.
(iv) according to the state of the fluid in the heat exchanger.

(i) Classification according to heat exchange process Heat exchangers may be of 'direct contact type' or of 'indirect contact type'. In direct contact type, two immiscible fluids come in direct contact with each other and exchange heat, e.g. air and water exchanging heat in a cooling tower. Indirect contact type can be further classified as 'recuperators' and 'regenerators'. Recuperators are most commonly used; here, the hot and cold fluids are separated from each other by a solid wall and heat is transferred from one fluid to the other across this wall. In regenerators, also called 'periodic flow heat exchangers', hot and cold streams alternately flow through a solid matrix (made of solid particles or wire mesh screens); during the 'hot blow', the matrix stores the heat given up by the hot stream and during 'cold blow', the stored heat is given up by the solid matrix to the cold stream. Sometimes, the solid matrix is made to rotate across fluid passages arranged side by side, so that the heat exchange process is 'continuous'.

(ii) Classification according to relative direction of hot and cold fluids If the hot and cold fluids flow parallel to each other, it is known as 'parallel flow' heat exchanger; if the two fluids flow opposite to each other, it is of

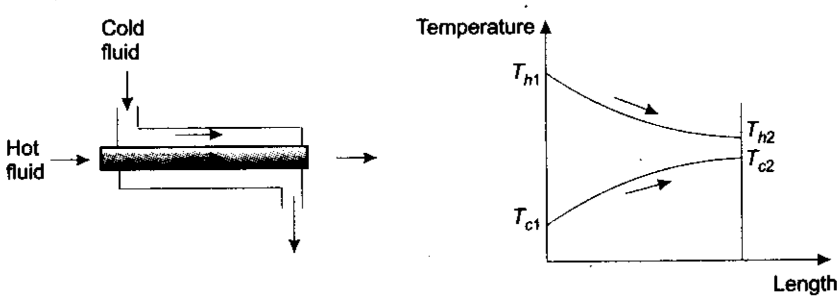


FIGURE 12.1(a) Parallel flow heat exchanger

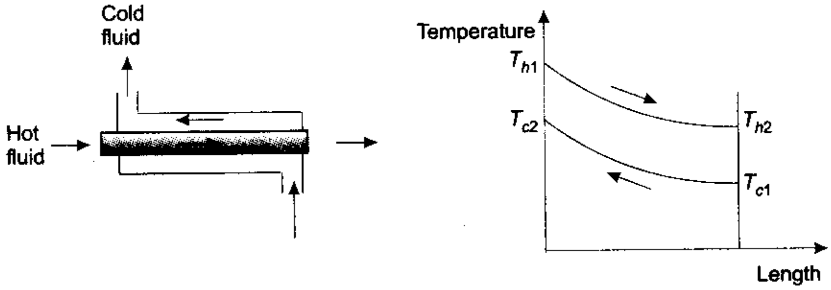


FIGURE 12.1(b) Counter-flow flow heat exchanger

'counter-flow' type. If the fluids flow perpendicular to each other, then, we have 'cross flow' type of heat exchanger. These three types of heat exchangers are shown schematically in Fig. 12.1.

Further, when a fluid is constrained to flow within a channel (such as a tube), the fluid is said to be 'unmixed'; otherwise, it is 'mixed'. In Fig. 12.1 (c), hot fluid is unmixed since it flows constrained within the tubes, whereas the cold fluid is perfectly mixed as it flows through the heat exchanger. In Fig. 12.1 (d), both the cold and hot fluids are constrained to flow within the tubes and therefore, both the fluids are unmixed.

(iii) Classification according to constructional features Basically, there are three types: (a) concentric tubes type (b) shell and tube type, and (c) compact heat exchangers.

In concentric tubes type of heat exchanger, one tube is located inside another; one fluid flows through the inside tube and the other fluid flows in the annular space between the tubes. Fluids may flow parallel to each other as shown in Fig. 12.1 (a), or they may flow in opposite directions, as shown in Fig. 12.1 (b).

Shell and tube type of heat exchanger is very popular in industry because of its reliability and high heat transfer effectiveness. Here, one of the fluids flows within a bundle of tubes placed within a shell. And, the other fluid flows through the shell over the surfaces of the tubes. Suitable baffles are provided within the shell to make the shell fluid change directions and provide good turbulence, so that heat transfer coefficient is increased.

Fig. 12.2 shows a schematic diagram of a typical shell and tube heat exchanger.

Fig. 12.2 is an example of two tube pass and one shell pass heat exchanger, i.e. flow passes through the tubes twice in opposite directions, and shell fluid passes through the shell once. Other flow arrangements are also used, such as: one shell pass + two, four or six tube passes; two shell passes and four, eight, twelve, etc. tube passes.

Compact heat exchangers are special purpose heat exchangers which provide very high surface area per cubic metre of volume, known as 'area density'. According to usually accepted norms, a 'compact heat exchanger' has an area density of 700 m^2/m^3 or more. These are generally used for gases, since usually gas side heat transfer coefficient is small and therefore, it is needed to provide larger areas. Compact heat exchangers are of plate-fin type or tube-fin type. A typical example of a plate-fin type of compact heat exchanger is shown in Fig. 12.3.

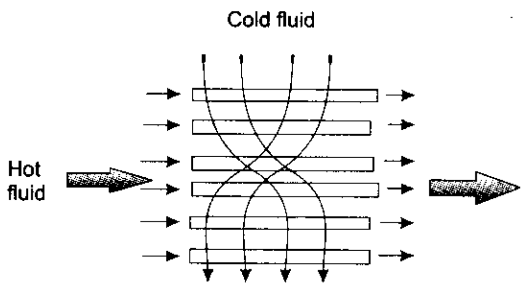


FIGURE 12.1(c) Cross-flow heat exchanger, cold fluid 'mixed', hot fluid 'unmixed'

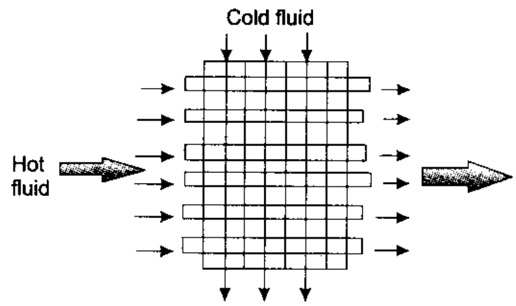


FIGURE 12.1(d) Cross-flow heat exchanger, both cold and hot fluids 'unmixed'

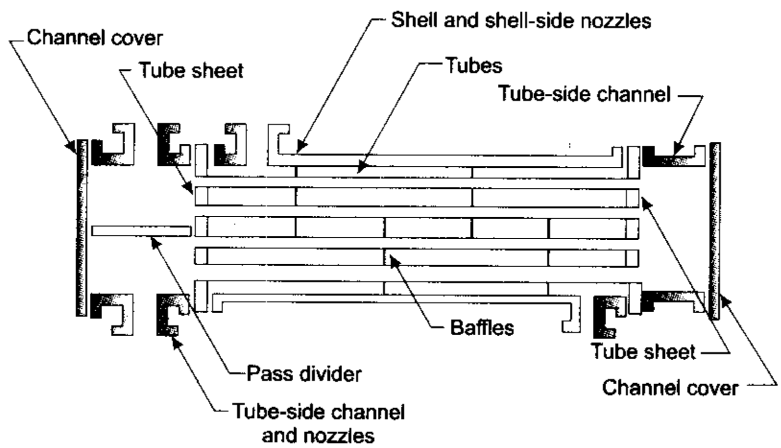


FIGURE 12.2 Diagram of a typical (fixed tube sheet) shell-and-tube heat exchanger

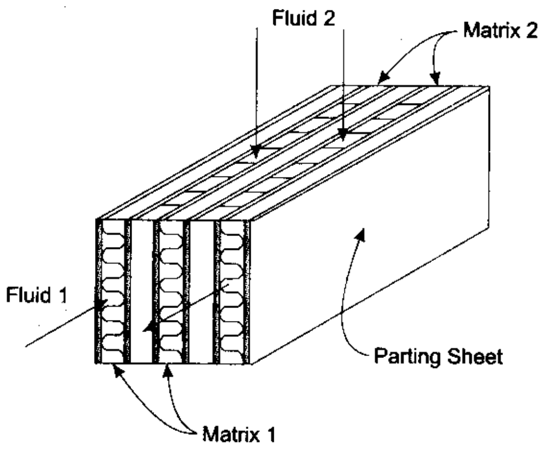


FIGURE 12.3 Section of a plate-fin heat exchanger

(iv) Classification according to state of the fluid In all the types of heat exchangers discussed above, both the fluids changed their temperature along the length of heat exchanger. But, this need not be the case always. A heat exchanger may be used to condense a fluid in which case the condensing fluid will be at a constant temperature throughout the length of the heat exchanger, while the other (cold) fluid will increase in temperature as it passes through the heat exchanger, absorbing the latent heat of condensation released by the condensing fluid. Such a heat exchanger is called a 'Condenser'. If, on the other hand, one of the fluids evaporates in a heat exchanger, tempera-ture of this fluid will remain constant throughout the length of heat exchanger, whereas the temperature of the other fluid, which supplies the latent heat of evaporation to the evaporating fluid, goes on decreasing along the length of the heat exchanger. Such a heat exchanger is called an 'Evaporator'.

It is interesting to compare the surface area-to-volume ratios of different types of heat exchangers. See Table 12.1:

TABLE 12.1 Surface area-to-volume ratios of different heat exchangers

Type of HX	Hydraulic diameter (mm)	Surface-area/Volume, (m^2/m^3)
Plain tube, shell-and-tube	40 to 6	60 – 600
Plate heat exchangers	20 to 10	180 – 350
Strip fin and louvred fin heat exchangers	10 to 0.5	350 – 7100
Automotive radiators	5 to 2.5	710 – 1500
Cryogenic-heat exchangers	3.7 to 1.7	1000 – 2500
Gas turbine rotary regenerators	1.2 to 0.5	3000 – 7100
Matrix types, wire screen, sphere bed, corrugated sheets	2.5 to 0.2	1500 – 18000
Human lungs	0.2 to 0.15	18000 – 25000

In Table 12.1, hydraulic diameter of the flow passage is also given; note that smaller the hydraulic diameter, larger is the ratio of surface area-to-volume. Note that the human lungs have the largest of all surface area-to-volume ratios.

12.3 Overall Heat Transfer Coefficient

'Overall heat transfer coefficient', was first introduced in Chapter 4. So, the reader may please refer back to Chapter 4 to refresh memory.

In most of the practical cases of heat exchangers, temperature of the hot fluid (T_a) and that of the cold fluid (T_b) are known; then we would like to have the heat transfer given by a simple relation of the form

$$Q = U \cdot A \cdot (T_a - T_b) = U \cdot A \cdot \Delta T \qquad \text{...(4.21)}$$

where, Q is the heat transfer rate (W), A is the area of heat transfer perpendicular to the direction of heat transfer, and $(T_a - T_b) = \Delta T$ is the overall temperature difference between the temperature of hot fluid (T_a) and that of the cold fluid (T_b).

In a normally used recuperative type of heat exchanger, the hot and cold fluids are separated by a solid wall. This may be a flat type of wall (as in the case of plate-fin type of heat exchangers), or, more often, a cylindrical wall (as in the case of a tube-in-tube type of heat exchangers). See Fig. 12.4.

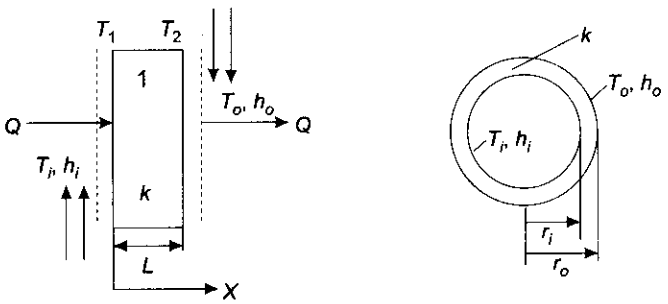
Recall from Chapter 4 that, in general, the overall heat transfer coefficient is related to the total thermal resistance of the system, as follows:

$$U = \frac{1}{A \cdot \Sigma R_{th}} \ \text{W/(m}^2\text{C).} \qquad \text{...(4.23)}$$

Therefore, the task of finding the overall heat transfer coefficient reduces to finding out the total thermal resistance of the system.

For plane wall:

Remember that for a plane wall, thermal resistance is $L/(k.A)$, and convective resistance is $1/(h.A)$, and since the resistances are in series, we get:



(a) Plane wall (b) Cylindrical wall

FIGURE 12.4 Heat exchanger walls

$$U = \frac{1}{A \cdot \left(\frac{1}{h_i \cdot A} + \frac{L}{k \cdot A} + \frac{1}{h_o \cdot A} \right)}$$

i.e.

$$U = \frac{1}{\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}} \text{ W/(m}^2\text{C).} \quad ...(12.1)$$

Now, if the thermal resistance of the wall is negligible compared to other resistances, we get:

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} \text{ W/(m}^2\text{C).} \quad ...(12.2)$$

For cylindrical wall:

Remember that for a cylindrical wall, thermal resistance is:

$$\frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L}$$

and, convective resistance is $1/(h.A)$ and the resistances are in series. However, the area to be considered has to be specified since the inner surface area and the outer surface area of the cylinder are different. Now, we have, the general relation for U:

$$U = \frac{1}{A \cdot \Sigma R_{th}} \text{ W/(m}^2\text{C).} \quad ...(4.23)$$

i.e.

$$U \cdot A = \frac{1}{\Sigma R_{th}}$$

We can also write:

$$U_i \cdot A_i = Uo \cdot Ao = \frac{1}{\Sigma R_{th}} \quad ...(12.3)$$

Therefore, referred to outer surface area, U becomes:

$$Uo \cdot Ao = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o \cdot A_o}} \quad ...(12.4)$$

Now, for a cylindrical system, we have:

$$A_i = 2 \cdot \pi \cdot r_i \cdot L$$

and,

$$A_o = 2 \cdot \pi \cdot r_o \cdot L$$

Then,

$$U_o = \frac{1}{\frac{A_o}{h_i \cdot A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} \cdot A_o + \frac{1}{h_o \cdot A_o} \cdot A_o}$$

i.e.

$$U_o = \frac{1}{\frac{1}{h_i} \cdot \left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{k}\right) \cdot \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}} \quad ...(12.5)$$

Similarly, referred to inner surface area, U becomes:

$$U_i \cdot A_i = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o \cdot A_o}} \quad \text{...(12.6)}$$

and,

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} \cdot A_i + \frac{1}{h_o \cdot A_o} \cdot A_i}$$

i.e.

$$U_i = \frac{1}{\frac{1}{h_i} + \left(\frac{r_i}{k}\right) \cdot \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} \cdot \left(\frac{r_i}{r_o}\right)} \quad \text{...(12.7)}$$

Again, if the thermal resistance of the wall is negligible compared to other resistances, (i.e. high value of thermal conductivity, k), or, wall thickness of the tube is very small (i.e. $(r_i/r_o) \approx 1$), we get:

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} \ \text{W/(m}^2\text{C)} \quad \text{...(12.8)}$$

Note that Eq. 12.8 is the same as Eq. 12.2. For many practical situations, this simple equation gives a quick estimate of overall heat transfer coefficient, U. Observe from Eq. 12.2 or 12.8 that the value of U is controlled by the smaller of the two heat transfer coefficients, h_i and h_o. Therefore, aim of the designer should be to focus on the smaller of the two heat transfer coefficients and improve it, if possible. For example, in a gas-to-liquid heat exchanger, heat transfer coefficient is generally smaller on the gas side, and, therefore, the gas side heat transfer coefficient controls the final value of overall heat transfer coefficient. So, one tries to improve the heat transfer coefficient on the gas side by providing fins on the gas side surface. If fins are provided on a particular surface, then the total heat transfer area on that surface is:

$$A_{\text{total}} = A_{\text{fin}} + A_{\text{unfinned}} \quad \text{...(12.9)}$$

where, A_{fin} is the surface area of the fins and A_{unfinned} is the area of the un-finned portion of the tube.

For short fins of a material of high thermal conductivity, since there is practically no temperature drop along the length we can use the value of total area as given by Eq. 12.9 to calculate the convection resistance on the finned surface. However, for long fins where there is a temperature drop along the length of fin, we should use the total or effective area, given by:

$$A_{\text{total}} = A_{\text{unfinned}} + \eta_{\text{fin}} \cdot A_{\text{fin}} \quad \text{...(12.10)}$$

where, η_{fin} is the 'fin efficiency'. Sometimes, an overall surface efficiency' η_o is used. η_o is defined as:

$$\eta_o \cdot A_{\text{total}} = A_{\text{unfinned}} + \eta_{\text{fin}} \cdot A_{\text{fin}}$$

i.e. η_o tells us how much of the total surface area is really effective in transferring heat.

Then, since the effective surface area is also equal to the unfinned area plus the effective area of fin, we can get an expression for overall surface efficiency as follows:

$$\eta_o \cdot A_{\text{total}} = (A_{\text{total}} - A_{\text{fin}}) + \eta_{\text{fin}} \cdot A_{\text{fin}}$$

i.e.

$$\eta_o = 1 - \frac{A_{\text{fin}}}{A_{\text{total}}} + \frac{\eta_{\text{fin}} \cdot A_{\text{fin}}}{A_{\text{total}}}$$

i.e.

$$\eta_o = 1 - \frac{A_{\text{fin}}}{A_{\text{total}}} \cdot (1 - \eta_{\text{fin}}). \quad \text{...(12.11)}$$

Then, while determining U, we should use $\eta_o \cdot A_{\text{total}}$ for the finned surface, whether it is inner surface area, outer surface area or both.

For example, if the outer surface of the tube is finned (which is usually the case), with a fin efficiency of η_{fin}, we write, neglecting the thermal resistance of tube material:

$$U_i \cdot A_i = U_o \cdot A_o = \frac{1}{\Sigma R_{th}} = \frac{1}{\dfrac{1}{h_i \cdot A_i} + \dfrac{1}{h_o \cdot (A_{unfinned} + \eta_{fin} \cdot A_{fin})_{outer\_surface}}} \quad ...(12.12)$$

Instead, if the total (i.e. unfinned + finned) surface area and the overall surface efficiency (η_o) is given for the outer surface, Eq. 12.12 can be written as:

$$U_i \cdot A_i = U_o \cdot A_o = \frac{1}{\Sigma R_{th}} = \frac{1}{\dfrac{1}{h_i \cdot A_i} + \dfrac{1}{h_o \cdot (\eta_o \cdot A_{total})_{outer\_surface}}} \quad ...(12.13)$$

Typical values of overall heat transfer coefficients are given in Table 12.2:

TABLE 12.2 Typical values of overall heat transfer coefficients

Type of HX	U (W/m²C)
Water-to-water	850 – 1700
Water-to-oil	100 – 350
Water-to-gasoline or kerosene	300 – 1000
Feed water heaters	1000 – 8500
Steam-to-light fuel oil	200 – 400
Steam-to-heavy fuel oil	50 – 200
Steam condenser	1000 – 6000
Freon condenser (water cooled)	300 – 1000
Ammonia condenser (water cooled)	800 – 1400
Alcohol condenser (water cooled)	250 – 700
Gas-to-gas	10 – 40
Water-to-air in finned tubes (water in tubes)	30 – 60 (based on water side surface area)
Steam-to-air in finned tubes (steam in tubes)	400 – 4000 (based on steam side surface area)

Fouling factors Note that above analysis was for clean heat transfer surfaces. However, with passage of time, the surfaces become 'dirty' because of scaling, deposits, corrosion, etc. This results in a reduction in heat transfer coefficient since the scale offers a thermal resistance to heat transfer. Fouling may be categorized as follows:

(i) due to scaling or precipitation
(ii) due to deposits of finely divided particulates
(iii) due to chemical reaction
(iv) due to corrosion
(v) due to attachments of algae or other biological materials
(vi) due to crystallization on the surface by subcooling.

Effect of fouling is accounted for by a term called, 'Fouling factor', (or, 'dirt factor'), defined as:

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \ \text{m}^2\text{K/W} \quad ...(12.14)$$

R_f is zero for a new heat exchanger. R_f for a fouled heat exchanger cannot be 'calculated' theoretically, but has to be determined experimentally by finding out the heat transfer coefficients for a 'clean' heat exchanger and a 'dirty' heat exchanger of identical design, operating under identical conditions.

While taking into account the effect of fouling, the 'fouling resistance' (= R_f/area) should be added to the other thermal resistances. For example, for a tube, we can write:

$$U_i \cdot A_i = U_o \cdot A_o = \frac{1}{\Sigma R_{th}} = \frac{1}{\dfrac{1}{h_i \cdot A_i} + \dfrac{R_{fi}}{A_i} + \dfrac{\ln\left(\dfrac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \dfrac{1}{h_o \cdot A_o} + \dfrac{R_{fo}}{A_o}} \quad ...(12.15)$$

where, R_{fi} and R_{fo} are the fouling factors for the inside and outside surfaces, respectively, and L is the length of tube. From Eq. 12.15, U_i or U_0 can easily be calculated.

Fouling factor depends on flow velocity and operating temperature; fouling increases with decreasing velocity and increasing temperature.

Based on experience, Tubular Exchanger Manufacturers' Association (TEMA) have given suggested values of fouling factors. Some of these values are given in Table 12.3:

TABLE 12.3 Fouling factors for industrial fluids (TEMA, 1988)

Fluid	R_f (m^2C/W)
LIQUIDS:	
Fuel oil	0.00088
Quench oil	0.0007
Transformer oil	0.00018
Hydraulic fluid	0.000238
Molten salts	0.000119
Industrial organic heat transfer media	0.000119
Refrigerant liquids	0.00018
Caustic solutions	0.000476
Vegetable oils	0.000715
Gasoline, naptha, light distillates, kerosene	0.000238
Light gas oil	0.000476
Heavy gas oil	0.000715
GASES & VAPOURS:	
Solvent vapours	0.000238
Acid gases	0.000238
Natural gas	0.000238
Air	0.000119 – 0.000238
Flue gases	0.000238 – 0.000715
Steam (sat., oil free)	0.000119 – 0.000357
WATER:	
River water, sea water, distilled water, boiler feed water:	
Below 50 deg.C	0.0001
Above 50 deg.C	0.0002

Example 12.1. Water at a mean temperature of $T_m = 90°C$ and a mean velocity of $u_m = 0.10$ m/s flows inside a 2.5 cm ID, thin-walled copper tube. Outer surface of the tube dissipates heat to atmospheric air at $T_a = 20°C$, by free convection. Calculate the tube wall temperature, overall heat transfer coefficient and heat loss per metre length of tube. Use following simplified expression for air to determine heat transfer coefficient by free convection:

$$h_a = 1.32 \cdot \left(\frac{T_s - T_a}{D} \right)^{0.25}$$

Solution.

Data:

$T_m := 90°C$ $\quad u_m := 0.1$ m/s $\quad D := 0.025$ m $\quad T_a := 20°C$

Properties of water at mean temperature of 90°C:

$\rho := 965.3$ kg/m^3 $\quad k := 0.675$ W/(mC) $\quad \mu := 0.315 \times 10^{-3}$ kg/(ms) $\quad Pr := 2.22$

We need to calculate the heat transfer coefficients for the inner and outer surfaces:

For the water side (i.e. inner surface):

We have: $$Re := \frac{D \cdot u_m \cdot \rho}{\mu}$$ *(Reynolds number)*

i.e. $$Re = 7.66 \times 10^3 > 4000$$ *(therefore, turbulent)*

Using Ditus–Boelter equation to determine heat transfer coefficient for inside surface:

$$Nu := 0.023 \cdot Re^{0.8} \cdot Pr^{0.3}$$

i.e. $Nu = 37.417$ *(Nusselts number)*

Therefore, $h_i := Nu \cdot \frac{k}{D}$

i.e. $h_i = 1.01 \times 10^3 \text{ W/(m}^2\text{C)}$ *(inside surface heat transfer coefficient)*

For the air side (i.e. outer surface):

Approximate, value of film temperature for air:

$$T_f := \frac{90+20}{2}$$

i.e. $T_f = 55°C$

Properties of air at film temperature of 55°C:

$\rho := 1.076 \text{ kg/m}^3$ $k := 0.0283 \text{ W/(mC)}$ $\mu := 1.99 \times 10^{-5} \text{ kg/(ms)}$ $Pr := 0.708$

Then, free convection heat transfer coefficient for outer surface is given by:

$$h_o = 1.32 \cdot \left(\frac{T_s - T_a}{D}\right)^{0.25}$$

i.e. $$h_o = 1.32 \cdot \left(\frac{T_s - 20}{0.025}\right)^{0.25}$$

i.e. $h_o = 3.32 \cdot (T_s - 20)^{0.25}$ ...(a)

However, T_s is not known,

Applying overall energy balance, with $A_i = A_o$ for thin-walled tube:

$$h_i \cdot (T_i - T_s) = h_o \cdot (T_s - T_a)$$

Substituting for h_i and h_o

$$1010 \cdot (90 - T_s) = [3.32 \cdot (T_s - 20)^{0.25}] \cdot (T_s - 20)$$

i.e. $1010 \cdot (90 - T_s) = [3.32 \cdot (T_s - 20)^{1.25}]$

This equation may now be solved for T_s by trial and error.

But, with Mathcad, it is easily solved using solve block. Start with a guess value of T_s, then, after typing 'Given' write down the constraint, and then, typing 'Find (T_s)' gives the value of T_s immediately:

$T_s := 40°C$ *(guess value)*

Given

$$1010 \cdot (90 - T_s) = [3.32 \cdot (T_s - 20)^{1.25}]$$

$$\text{Find}(T_s) := 89.342$$

i.e. $T_s := 89.342°C$ *(tube surface temperature)*

Then, h_o is calculated from Eq. a:

$h_o := 3.32 \cdot (T_s - 20)^{0.25}$ ...(a)

i.e. $h_o = 9.58 \text{ W/(m}^2\text{C)}$ *(outside surface heat transfer coefficient)*

and, overall heat heat transfer coefficient, U:

$$U := \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

i.e. $U = 9.49 \text{ W(m}^2\text{C)}$ *(overall heat transfer coefficient)*

Note that overall heat transfer coefficient is nearly equal to h_o. As commented earlier, since $h_i >> h_o$, overall heat transfer coefficient is conrolled by h_o.

Heat loss per metre length of tube:

$$Q := U \cdot (\pi \cdot D \cdot 1) \cdot (T_s - T_a) \text{ W/m}$$

i.e. $Q = 51.686$ **W/m.**

Example 12.2. In Exaple 12.1, if we desire to increase the value of overall heat transfer coefficient U, the obvious choice is to focus on the air-side, since the air side heat transfer coefficient is the lower of the inside and outside heat transfer coefficients. Let us increase the area on the air side by providing 8 numbers of radial fins of rectangular cross section, 2 mm thick and 20 mm height. Material of fins is the same as that of the tube, i.e. copper (k = 380 W/(mK)).

Then, determine the overall heat transfer coefficient and the rate of heat transfer.

Solution.

Data:

$T_m := 90°C$ $u_m := 0.1$ m/s $D := 0.025$ m $T_a := 20°C$ $Ts := 89.342°C$ $h_i := 1010$ W/(m^{2}C)

$h_o := 9.58$ W/(m^{2}C) $L := 0.02$ m (fin height) $W := 1$ m (fin width = length of cyl.)

$t := 0.002$ m (fin thickness) $N := 8$ (No. of fins) $k := 380$ W/(mK)

$A_i := \pi \cdot D \cdot 1$ m^2/metre i.e. $A = 0.079$ m^2/metre

Overall heat transfer coefficient Ui referred to the inside surface:

Neglecting the thermal resistance of tube wall, we write:

$$U_i \cdot A_i = \frac{1}{\Sigma R_{th}} = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{1}{h_o \cdot (A_{unfinned} + \eta_{fin} \cdot A_{fins})}} \quad \text{...(a)}$$

First term in the denominator in RHS is the thermal resistance due to film coefficient on the inside, and the second term is the thermal resistance of the film coefficient on the outside.

Un-finned surface (or the base surface) on the outside is at the wall temperature and is fully effective for heat transfer whereas the finned surface is not fully effective because of temperature drop along the length of fins; therefore, effective area of fins is obtained by multiplying the total area of fins by the fin effectiveness, η_{fin}.

Therefore, we need to find out the fin efficiency.

Fin efficiency:

For a rectangular fin with adiabatic tip, the fin efficiency is given by:

$$\eta_{fin} = \frac{\tanh(m \cdot L)}{m \cdot L} \quad \text{...(b)}$$

where, $m = \sqrt{\frac{h_o \cdot P}{k \cdot A_c}}$ 1/m *(fin parameter)*

$P = 2 \cdot (W + t)$ *(perimeter, W = width of fin = 1 m)*

$A_c = W \cdot t$ *(area of cross section of fin)*

Then, $\frac{P}{A_c} = \frac{2 \cdot (W + t)}{W \cdot t} = \frac{2}{t}$ *(for t << W)*

Therefore, $m := \sqrt{\frac{2 \cdot h_o}{k \cdot t}}$

i.e. $m = 5.021$ 1/m

and, $m \cdot L = 0.1$

Then, from Eq. b, we get:

$$\eta_{fin} := \frac{\tanh(m \cdot L)}{m \cdot L}$$

i.e. $\eta_{fin} = 0.997$ *(fin efficiency)*

Areas:

$A_{unfinned} := \pi \cdot (D - N \cdot t) \cdot W$

i.e. $A_{unfinned} = 0.028$ m^2 *(unfinned or prime (base) area)*

$A_{fins} := N \cdot (2 \cdot W \cdot L)$ m^2 *(finned area of N fins (both upper and lower side of fins considered)*

i.e. $A_{fins} = 0.32$ m^2

Now, $A_i := \pi \cdot D \cdot W$ m^2 *(inside surface area per metre length)*

i.e. $A_i = 0.079$ m^2

Therefore, overall heat transfer coefficient U_i referred to the inside surface:

We have:
$$U_i \cdot A_i = \frac{1}{\Sigma R_{th}} = \frac{1}{\frac{1}{h_i \cdot A_i} + \frac{1}{h_o \cdot (A_{unfinned} + \eta_{fin} \cdot A_{fins})}} \quad \text{...(a)}$$

i.e. $U_i \cdot A_i = 3.192$

and, $U_i := \frac{3.192}{A_i}$

i.e. $U_i = 40.642$ W/(m^{2}C) *(overall heat transfer coefficient referred to inside area)*

Note: compare this to the earlier U value of 9.49 W/(m^{2}C); there is great improvement in value of U by providing fins.

Heat loss per metre length of tube:

$$Q := Ui \cdot (\pi \cdot D \cdot 1) \cdot (T_s - T_a) \text{ W/m}$$

i.e. $$Q = 221.34 \text{ W/m}.$$

Note: Compare this to the earlier Q value of 51.686 W/m; this substantial improvement in value of Q is the result of providing fins.

Also, note that in the above analysis, we assumed that the outside heat transfer coefficient h_o is the same for the un-finned surface as well as for the finned surfaces.

Example 12.3. A shell and tube counter-flow heat exchanger uses copper tubes (k = 380 W/(mC)), 20 mm ID and 23 mm OD. Inside and outside film coefficients are 5000 and 1500 W/(m^{2}C), respectively. Fouling factors on the inside and outside may be taken as 0.0004 and 0.001 m^{2}C/W respectively. Calculate the overall heat transfer coefficient based on: (i) outside surface, and (ii) inside surface.

Solution.

Data:

$D_i := 0.020$ m $\quad D_o := 0.023$ m $\quad L := 1$ m $\quad k := 380$ W/(mK) $\quad h_i := 5000$ W/(m^{2}C) $\quad h_o := 1500$ W/(m^{2}C)

$R_{fi} := 0.0004$ m^{2}C/W (*fouling factor on inside surface*) $\quad R_{fo} := 0.001$ m^{2}C/W (*fouling factor on outside surface*)

Heat transfer areas:

$$A_o := \pi \cdot D_o \cdot L \text{ m}^2\text{/metre} \qquad \textit{(inside surface area)}$$

i.e. $$A_o = 0.07226 \text{ m}^2\text{/metre}$$

and, $$A_i := \pi \cdot D_i \cdot L \text{ m}^2\text{/metre} \qquad \textit{(inside surface area)}$$

i.e. $$A_i = 0.06283 \text{ m}^2\text{/metre}$$

Overall heat transfer coefficient:

We have:

$$U_i \cdot A_i = U_o \cdot A_o = \frac{1}{\Sigma R_{th}} = \frac{1}{\dfrac{1}{h_i \cdot A_i} + \dfrac{R_{fi}}{A_i} + \dfrac{\ln\left(\dfrac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \dfrac{1}{h_o \cdot A_o} + \dfrac{R_{fo}}{A_o}} \qquad \text{...(12.15)}$$

In the denominator of RHS of Eq. 12.15 above, we have the various thermal resistances, as follows:

first term → convective film resistance on the inside surface = 3.1831×10^{-3} C/W

second term → fouling resistance on the inside surface = 6.3662×10^{-3} C/W

third term → conductive resistance of the tube wall = 5.854×10^{-5} C/W

fourth term → convective film resistance on the outside surface = 9.2264×10^{-3} C/W

fifth term → fouling resistance on the outside surface = 0.01384 C/W.

Note the relative magnitude of fouling resistances, as compared to other resistances. As expected, conductive resistance of the tube wall (of copper, which is a good conductor) is the smallest of all.

Calculating the RHS, we get:

$$\frac{1}{\dfrac{1}{h_i \cdot A_i} + \dfrac{R_{fi}}{A_i} + \dfrac{\ln\left(\dfrac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \dfrac{1}{h_o \cdot A_o} + \dfrac{R_{fo}}{A_o}} = 30.606$$

i.e. $$U_i \cdot A_i = U_o \cdot A_o = 30.606$$

Therefore, $$U_i := \frac{30.606}{A_i}$$

i.e. $$U_i = 487.11 \text{ W/(m}^2\text{C)} \qquad \textbf{\textit{(Overall heat transfer based on inside surface.)}}$$

and, $$U_o := \frac{30.606}{A_o}$$

i.e. $$U_o = 423.574 \text{ W/(m}^2\text{C)} \qquad \textbf{\textit{(Overall heat transfer coefficient based on outside surface.)}}$$

Comments:

'Fouling' affects the value of overall heat transfer coefficient and therefore, the size (or area) of the heat exchanger adversely.

If the fouling resistances were not included, we should have obtained the following values for the overall heat transfer coefficient:

RHS of Eq. 12.15, deleting the fouling resistances, will become:

$$\frac{1}{\frac{1}{h_i \cdot A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o \cdot A_o}} = 80.205$$

i.e. $U_i \cdot A_i = U_o \cdot A_o = 80.205$

Therefore, $U_i := \frac{80.205}{A_i}$

i.e. $U_i = 1.277 \times 10^3$ W/(m^{2}C) *(overall heat transfer coefficient based on inside surface.)*

And, $U_o := \frac{80.205}{A_o}$

i.e. $U_o = 1.11 \times 10^3$ W/(m^{2}C) *(Overall heat transfer coefficient based on outside surface.)*

i.e. U_o and U_i with no fouling are about 2.62 times the corresponding values when fouling resistances are included. Therefore, it is advisable to include the effect of fouling, if practicable, at the design stage.

12.4 The LMTD Method for Heat Exchanger Analysis

Basically, a complete design of a heat exchanger is a huge topic which involves an analysis of:

(i) Thermal aspects (i.e. temperatures of fluids at inlet/exit, rate of heat transfer, etc., off-design performance, etc.)

(ii) Hydrodynamic aspects (i.e. pressure drops in the flow channels)

(iii) Structural aspects (mechanical design and structural design).

However, here we shall consider only the thermal analysis aspects.

12.4.1 Parallel Flow Heat Exchanger

Consider a double pipe, parallel flow heat exchanger, in which a hot fluid and a cold fluid flow parallel to each other, separated by a solid wall. Hot fluid enters at a temperature of T_{h1} and leaves the heat exchanger at a temperature of T_{h2}; cold fluid enters the heat exchanger at a temperature of T_{c1} and leaves at a temperature of T_{c2}. This situation is shown in Fig. 12.5.

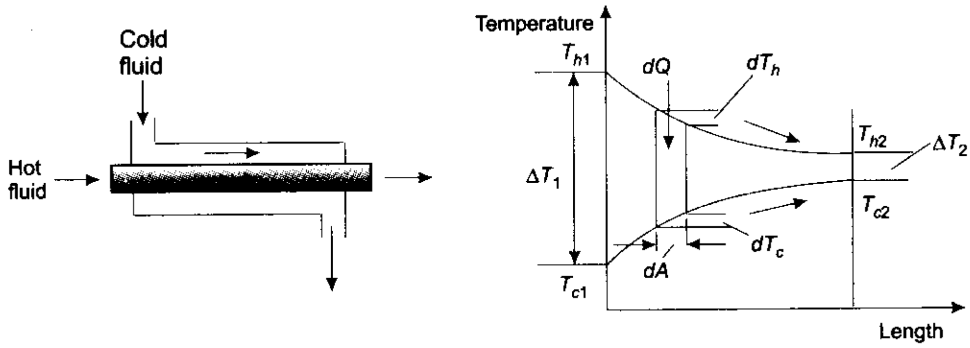


FIGURE 12.5 Parallel flow heat exchanger

We desire to get an expression for the rate of heat transfer in this heat exchanger in the following form:

$$Q = U \cdot A \cdot \Delta T_m \qquad \text{...(12.16)}$$

where, U = overall heat transfer coefficient

A = area for heat transfer (should be the same area on which U is based), and

ΔT_m = a mean temperature difference between the fluids.

Now, we make the following assumptions:

(i) U is considered as a constant throughout the length (or area) of the heat exchanger

(ii) Properties of fluids (such as specific heat) are also considered to be constant with temperature

(iii) Heat exchange takes place only between the two fluids and there is no loss of heat to the surroundings, i.e. perfect insulation of heat exchanger is assumed

(iv) Changes in potential and kinetic energy are negligible

(v) Temperatures of both the fluids remain constant (equal to their bulk temperatures) over a given cross section of the heat exchanger.

Area 'A' is constant for a given heat exchanger. However, we see from Fig. 12.5 that the temperature of the two fluids vary along the length (or area) of the heat exchanger, i.e. the temperature difference between the hot and cold fluids is not a constant along the length of the heat exchanger, but varies along the length. Our aim is to find out the appropriate 'mean temperature difference (ΔT_m)' between the hot and cold fluids, so that Eq. 12.16 can be applied. We proceed as follows:

Consider an elemental area dA of the heat exchanger. Then, by applying the First law, we can write:

Heat given up by the hot fluid = heat received by the cold fluid.

i.e.

$$dQ = -m_h \cdot C_{ph} \cdot dT_h = m_c \cdot C_{pc} \cdot dT_c \quad ...(12.17)$$

Here, the temperature of hot fluid decreases as the length increases. So, a negative sign is put in front of $m_h.C_{ph}.dT_h$, so that the heat transferred is a positive quantity.

Now, dQ for the elemental area dA, can also be expressed as:

$$dQ = U \cdot (T_h - T_c) \cdot dA \quad ...(12.18)$$

Now, from Eq. 12.17, we have:

$$dT_h = \frac{-dQ}{m_h \cdot C_{ph}}$$

and,

$$dT_c = \frac{dQ}{m_c \cdot C_{pc}}$$

where, m_h and m_c are the mass flow rates, and C_{ph} and C_{pc} are the specific heats of hot and cold fluids, respectively.

Therefore,

$$dT_h - dT_c = d(T_h - T_c) = -dQ \cdot \left(\frac{1}{m_h \cdot C_{ph}} + \frac{1}{m_c \cdot C_{pc}} \right) \quad ...(12.19)$$

Substituting for dQ from Eq. 12.18, we get:

$$d(T_h - T_c) = -U \cdot (T_h - T_c) \cdot dA \cdot \left(\frac{1}{m_h \cdot C_{ph}} + \frac{1}{m_c \cdot C_{pc}} \right)$$

i.e.

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U \cdot \left(\frac{1}{m_h \cdot C_{ph}} + \frac{1}{m_c \cdot C_{pc}} \right) \cdot dA \quad ...(12.20)$$

Integrating Eq. 12.20 between the inlet and exit of the heat exchanger (i.e. between conditions 1 and 2):

$$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} \right) = -U \cdot A \cdot \left(\frac{1}{m_h \cdot C_{ph}} + \frac{1}{m_c \cdot C_{pc}} \right) \quad ...(12.21)$$

Now, considering the total heat transfer rate for the entire heat exchanger, we have:

$$m_h \cdot C_{ph} = \frac{Q}{T_{h1} - T_{h2}}$$

and,

$$m_c \cdot C_{pc} = \frac{Q}{T_{c2} - T_{c1}}$$

Substituting in Eq. 12.21:

$$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} \right) = \frac{-U \cdot A}{Q} \cdot (T_{h1} - T_{h2} + T_{c2} - T_{c1})$$

i.e.
$$Q = U \cdot A \cdot \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln\left(\dfrac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right)} \qquad ...(12.22)$$

Now, comparing Eq. 12.22 and Eq. 12.16, we observe that:

$$\Delta T_m = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln\left(\dfrac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right)} \qquad ...(12.23)$$

Since this mean temperature difference varies in a logarithmic manner, it is called 'Logarithmic Mean Temperature Difference' or, simply LMTD.

So, we write:

$$\text{LMTD} = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln\left(\dfrac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right)} \qquad ...(12.24)$$

Now, note that $(T_{h2} - T_{c2})$ is the temperature difference at the exit and $(T_{h1} - T_{c1})$ is the temperature difference at the inlet of the heat exchanger. If we denote the temperature differences at the inlet and exit of the heat exchanger by ΔT_1 and ΔT_2, respectively, we can write:

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} \qquad ...(12.25)$$

We can state Eq. 12.25 in words as follows: LMTD is equal to the ratio of the difference between the greater and lower of the temperature differences at the two ends to the natural logarithm of the ratio between those temperature differences.

Equation for LMTD is easily remembered as follows:

$$\text{LMTD} = \frac{\text{GTD} - \text{LTD}}{\ln\left(\dfrac{\text{GTD}}{\text{LTD}}\right)} \qquad ...(12.26)$$

where,

GTD = 'greater (of the two) temperature difference', and

LTD = 'lower temperature difference'.

12.4.2 Counter-flow Heat Exchanger

Again, consider a double pipe, counter-flow heat exchanger, in which a hot fluid and a cold fluid flow in directions opposite to each other, separated by a solid wall. Hot fluid enters at a temperature of T_{h1} and leaves the heat exchanger at a temperature of T_{h2}; cold fluid enters the heat exchanger at a temperature of T_{c1} and leaves at a temperature of T_{c2}. This situation is shown in Fig. 12.6.

We desire to get an expression for the rate of heat transfer in this heat exchanger in the following form:

$$Q = U \cdot A \cdot \Delta T_m \qquad ...(12.16)$$

where,

U = overall heat transfer coefficient

A = area for heat transfer (should be the same area on which U is based), and

ΔT_m = a mean temperature difference between the fluids.

We see from Fig. 12.6 that the temperatures of the two fluids vary along the length (or area) of the heat exchanger, i.e. the temperature difference between the hot and cold fluids is not a constant along the length of the heat exchanger, but varies along the length. Our aim is to find out the appropriate 'mean temperature difference (ΔT_m)' between the hot and cold fluids, so that Eq. 12.16 can be applied. We proceed as follows, with the same assumptions as made for the analysis of parallel flow heat exchanger:

Consider an elemental area dA of the heat exchanger. Then, by applying the First law, we can write:

Heat given up by the hot fluid = heat received by the cold fluid.

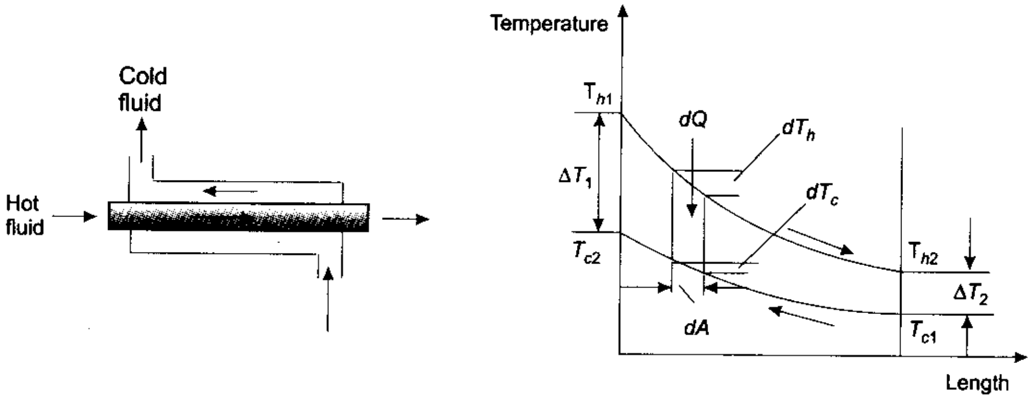


FIGURE 12.6 Counter-flow heat exchanger

i.e.

$$dQ = -m_h \cdot C_{ph} \cdot dT_h = -m_c \cdot C_{pc} \cdot dT_c \quad ...(12.27)$$

Here, the temperatures of both hot and cold fluids decrease as the length increases. So, negative sign is put in front of $m_h.C_{ph}.dT_h$ and $m_c.C_{pc}.dT_c$ so that the heat transferred is a positive quantity.

Now, dQ for the elemental area dA, can also be expressed as:

$$dQ = U \cdot (T_h - T_c) \cdot dA \quad ...(12.28)$$

Now, from Eq. 12.27, we have:

$$dT_h = \frac{-dQ}{m_h \cdot C_{ph}}$$

and,

$$dT_c = \frac{-dQ}{m_c \cdot C_{pc}}$$

where, m_h and m_c are the mass flow rates, and C_{ph} and C_{pc} are the specific heats of hot and cold fluids, respectively.

Therefore,

$$dT_h - dT_c = d(T_h - T_c) = -dQ \cdot \left(\frac{1}{m_h \cdot C_{ph}} - \frac{1}{m_c \cdot C_{pc}} \right) \quad ...(12.29)$$

Substituting for dQ from Eq. 12.28, we get:

$$d(T_h - T_c) = -U \cdot (T_h - T_c) \cdot dA \cdot \left(\frac{1}{m_h \cdot C_{ph}} - \frac{1}{m_c \cdot C_{pc}} \right)$$

i.e.

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U \cdot \left(\frac{1}{m_h \cdot C_{ph}} - \frac{1}{m_c \cdot C_{pc}} \right) \cdot dA \quad ...(12.30)$$

Integrating Eq. 12.30 between the inlet and exit of the heat exchanger (i.e. between conditions 1 and 2):

$$\ln\left(\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}} \right) = -U \cdot A \cdot \left(\frac{1}{m_h \cdot C_{ph}} - \frac{1}{m_c \cdot C_{pc}} \right) \quad ...(12.31)$$

Now, considering the total heat transfer rate for the entire heat exchanger, we have:

$$m_h \cdot C_{ph} = \frac{Q}{T_{h1} - T_{h2}}$$

and,

$$m_c \cdot C_{pc} = \frac{Q}{T_{c2} - T_{c1}}$$

Substituting in Eq. 12.31:

$$\ln\left(\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}}\right) = \frac{-U \cdot A}{Q} \cdot (T_{h1} - T_{h2} - T_{c2} + T_{c1})$$

i.e.

$$Q = U \cdot A \cdot \frac{(T_{h2} - T_{c1}) - (T_{h1} - T_{c2})}{\ln\left(\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}}\right)} \qquad \text{...(12.32)}$$

Now, comparing Eq. 12.32 and Eq.12.16, we observe that:

$$\Delta T_m = \frac{(T_{h2} - T_{c1}) - (T_{h1} - T_{c2})}{\ln\left(\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}}\right)} \qquad \text{...(12.33)}$$

Note that this mean temperature difference varies in a logarithmic manner; so, it is called 'Logarithmic Mean Temperature Difference' or, simply LMTD.

So, we write:

$$\text{LMTD} = \frac{(T_{h2} - T_{c1}) - (T_{h1} - T_{c2})}{\ln\left(\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}}\right)} \qquad \text{...(12.34)}$$

Now, note that $(T_{h2} - T_{c1})$ is the temperature difference at the exit and $(T_{h1} - T_{c2})$ is the temperature difference at the inlet of the heat exchanger. If we denote the temperature differences at the inlet and exit of the heat exchanger by ΔT_1 and ΔT_2, respectively, we can write:

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} \qquad \text{...(12.35)}$$

Note that the LMTD expressions for the parallel flow and the counter-flow heat exchangers (i.e. Eqs. 12.25 and 12.35) are the same.

Again, equation for LMTD is easily remembered as follows:

$$\text{LMTD} = \frac{\text{GTD} - \text{LTD}}{\ln\left(\frac{\text{GTD}}{\text{LTD}}\right)} \qquad \text{...(12.35a)}$$

where

GTD = 'greater (of the two) temperature difference', and

LTD = 'lower temperature difference'

Comments:

(i) When $\Delta T_1 = \Delta T_2$: This is a special case, which can occur sometimes in the case of a counter-flow heat exchanger. Then, Eq. 12.35 reduces to a form 0/0, which is indeterminate. However, from physical considerations, $\Delta T_1 = \Delta T_2$ means that the temperature difference between the hot and cold fluids is equal throughout the heat exchanger. Therefore, obviously, the mean temperature difference between the two fluids is $\Delta T_1 = \Delta T_2$. (This can be proved mathematically also, by applying L'Hospital's rule).

(ii) LMTD for a counterflow heat exchanger is always greater than that for a parallel flow heat exchanger. This means that to transfer the same amount of heat, counterflow unit will require a smaller heat transfer surface as compared to a parallel flow unit. This is the reason why a counter-flow heat exchanger is usually preferred.

(iii) LMTD can easily be calculated when all the end temperatures of the fluids are known. Then, immediately, the heat transfer rate is determined from the Eq. 12.16, i.e. $Q = U.A.(\text{LMTD})$. Therefore, calculation of

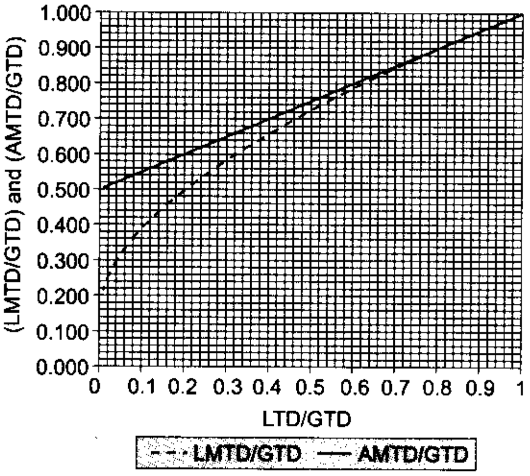


FIGURE 12.7 LMTD and AMTD for parallel and counter-flow HX

LMTD is an important step in the design of a heat exchanger. To facilitate quick calculation of LMTD, when both the end temperatures are known, following graph (Fig. 12.7) is provided. Here, (LMTD/GTD) is plotted against the ratio (LTD/GTD), where GTD = greater of the two end temperature differences, and LTD = lower of the two end temperature differences. First, calculate the ratio (LTD/GTD), and then, read (either from the graph or Table 12.4) the value of LMTD/GTD. Next, multiply this value by GTD to get LMTD.

In the same graph, the value of (AMTD/GTD) is also plotted, for comparison. Here, AMTD is the arithmetic mean temperature difference; AMTD = $(\Delta T_1 + \Delta T_2)/2$. It may be noted that for values beyond about $\Delta T_2/\Delta T_1 = 0.7$, AMTD and LMTD are almost the same, i.e. when $\Delta T_2/\Delta T_1 > 0.7$, it would suffice to use AMTD (which is easier to calculate) instead of LMTD. However, for lower values of $\Delta T_2/\Delta T_1$, LMTD has to be used.

Graph for LMTD shown above is also represented in tabular form (for better accuracy) in Table 12.4,

(iv) One term occurring in the derivation of LMTD shown above, is the product of mass flow rate and the specific heat of a fluid, i.e. $C = m.C_p$. Here, C is known as 'heat capacity rate' or, simply 'capacity rate' of that particular fluid. Thus, the capacity rates for hot and cold fluids are:

$$C_h = m_h \cdot C_{ph} \text{ W/C} \qquad \text{((12.36)...}\textit{capacity rate for hot fluid})$$

$$C_c = m_c \cdot C_{pc} \text{ W/C} \qquad \text{((12.37)...}\textit{capacity rate for cold fluid})$$

Then, the heat transfer rate is given by:

$$Q = C_h \cdot (T_{h1} - T_{h2}) \text{ W} \qquad \text{((12.38)...}\textit{for hot fluid})$$

$$Q = C_c \cdot (T_{c2} - T_{c1}) \text{ W} \qquad \text{((12.38)...}\textit{for cold fluid})$$

i.e. to transfer a given amount of heat, higher the heat capacity rate of a fluid, lower will be the temperature rise (or fall) of that particular fluid.

If the heat capacity rates of both the hot and cold fluids are equal, then, the total temperature drop of the hot fluid will be equal to the total temperature rise of the cold fluid. See Fig. 12.8 (a).

(v) When a fluid is condensing or boiling, its temperature is essentially constant, i.e. $T_{h1} = T_{h2}$ for a condensing fluid and $T_{c1} = T_{c2}$ for a boiling liquid. In other words, ΔT for the condensing or boiling fluid is zero. But, since a finite amount of heat is transferred, $(= m.h_{fg})$, we say that capacity rate of a condensing or boiling fluid tends to infinity. Temperature profiles for fluids in a heat exchanger when one of the fluids is condensing or boiling are shown in Fig. 12.8 (b) and (c), respectively. LMTD for both these cases is determined by the same procedure as for the parallel or counter-flow heat exchangers, i.e.

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} \qquad \text{...(12.35)}$$

Example 12.4. Furnace oil, flowing at a rate of 4000 kg/h, is heated from 10 to 20°C by hot water flowing at 75°C, with a velocity of 0.8 m/s, through a copper pipe 2.15 cm OD, 1.88 cm ID. Oil flows through annulus between copper and steel pipe of 3.35 cm OD and 3 cm ID. Find the length of counter-flow heat exchanger. Fluid properties are given.

Use Dittus–Boelter equation $Nu = 0.023.Re^{0.8}.Pr^{0.4}$.

Solution.

Data: $m_c := \dfrac{4000}{3600}$

i.e. $m_c = 1.111$ kg/s

$T_{c1} := 10°C \quad T_{c2} := 20°C \quad Th_1 := 75°C \quad V := 0.8$ m/s $\quad k_{cu} := 385$ W/(mK)

TABLE 12.4 LMTD/GTD for parallel flow and counter-flow HX
LMTD = Log mean temperature difference
GTD = Greater of two end temperature differences
LTD = Lower of two end temperature differences

LTD/GTD	LMTD/GTD	LTD/GTD	LMTD/GTD	LTD/GTD	LMTD/GTD
0.01	0.215	0.36	0.626	0.71	0.847
0.02	0.251	0.37	0.634	0.72	0.852
0.03	0.277	0.38	0.641	0.73	0.858
0.04	0.298	0.39	0.648	0.74	0.863
0.05	0.317	0.4	0.655	0.75	0.869
0.06	0.334	0.41	0.662	0.76	0.875
0.07	0.350	0.42	0.669	0.77	0.880
0.08	0.364	0.43	0.675	0.78	0.885
0.09	0.378	0.44	0.682	0.79	0.891
0.1	0.391	0.45	0.689	0.8	0.896
0.11	0.403	0.46	0.695	0.81	0.902
0.12	0.415	0.47	0.702	0.82	0.907
0.13	0.426	0.48	0.708	0.83	0.912
0.14	0.437	0.49	0.715	0.84	0.918
0.15	0.448	0.5	0.721	0.85	0.923
0.16	0.458	0.51	0.728	0.86	0.928
0.17	0.468	0.52	0.734	0.87	0.933
0.18	0.478	0.53	0.740	0.88	0.939
0.19	0.488	0.54	0.747	0.89	0.944
0.2	0.497	0.55	0.753	0.9	0.949
0.21	0.506	0.56	0.759	0.91	0.954
0.22	0.515	0.57	0.765	0.92	0.959
0.23	0.524	0.58	0.771	0.93	0.965
0.24	0.533	0.59	0.777	0.94	0.970
0.25	0.541	0.6	0.783	0.95	0.975
0.26	0.549	0.61	0.789	0.96	0.980
0.27	0.558	0.62	0.795	0.97	0.985
0.28	0.566	0.63	0.801	0.98	0.990
0.29	0.574	0.64	0.807	0.99	0.995
0.3	0.581	0.65	0.812	1	1.000
0.31	0.589	0.66	0.818		
0.32	0.597	0.67	0.824		
0.33	0.604	0.68	0.830		
0.34	0.612	0.69	0.835		
0.35	0.619	0.7	0.841		

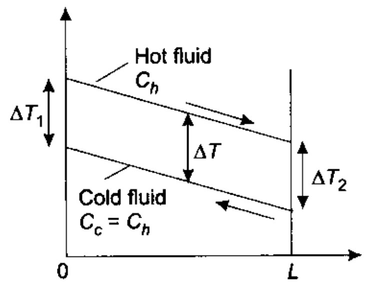


FIGURE 12.8 (a) Both fluids have same capacity rates

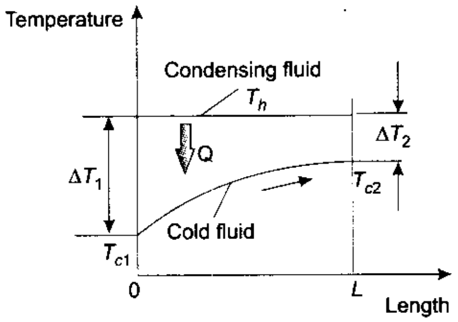


FIGURE 12.8 (b) One of the fluids condensing ($C_h \Rightarrow \infty$)

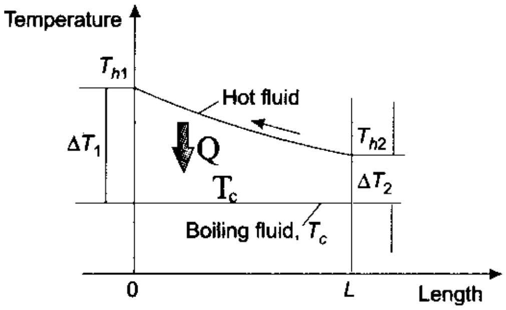


Figure 12.8 (c) One of the fluids boiling ($C_c \Rightarrow \infty$)

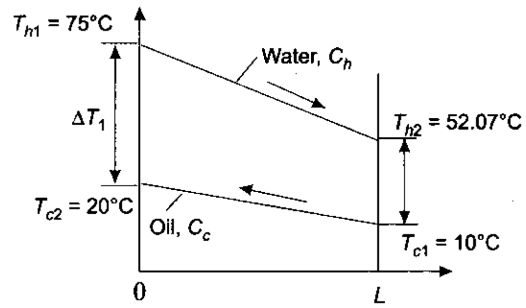


FIGURE Example 12.4 Counter-flow heat exchanger

Fluid properties:

Property	Water	Oil
C_p (kJ/kgK)	4.187	1.884
k (W/mK)	0.657	0.138
ν (m²/s)	4.187×10^{-7}	7.43×10^{-6}
ρ (kg/m³)	982	854

$Dh_i := 1.88 \times 10^{-2}$ m *(inside diameter of tube for hot fluid flow)*

$Dh_o := 2.15 \times 10^{-2}$ m *(outside diameter of tube for hot fluid flow)*

$Dc_i := 3 \times 10^{-2}$ m *(inside diameter of tube for cold fluid flow)*

$Dc_o := 3.35 \times 10^{-2}$ m *(outside diameter of tube for cold fluid flow)*

$C_{ph} := 4187$ J/(kgK) $\quad k_h := 0.657$ W/(mK) $\quad \nu_h := 4.18 \times 10^{-7}$ m²/s $\quad \rho_h := 982$ kg/m³ $\quad c_{pc} := 1884$ J/(kgK)

$k_c := 0.138$ W/(mK) $\quad \nu_c := 7.43 \times 10^{-6}$ m²/s $\quad \rho_c := 854$ kg/m³

$$Pr_h := \frac{c_{ph} \cdot (\nu_h \cdot \rho_h)}{k_h} \text{ i.e. } Pr_h = 2.616 \qquad Pr_c := \frac{c_{pc} \cdot (\nu_c \cdot \rho_c)}{k_c} \text{ i.e. } Pr_c = 86.626$$

Total heat transferred:

$$Q := m_c \cdot c_{pc} \cdot (Tc_2 - Tc_1) \text{ W}$$

i.e. $$Q = 2.093 \times 10^4 \text{ W}$$

Inside heat transfer coefficient:

$$A_i := \pi \cdot \frac{Dh_i^2}{4} \text{ m}^2$$ *(cross-sectional area of inside tube)*

i.e. $$A_i = 2.776 \times 10^{-4} \text{ m}^2$$ *(cross-sectional area of inside tube)*

and, $$m_h := A_i \cdot V \cdot \rho_h$$

i.e. $$m_h = 0.218 \text{ kg/s}$$ *(mass flow rate of hot fluid)*

Therefore,

$$G_h := \frac{m_h}{A_i}$$

i.e. $$G_h = 785.6 \text{ kg/(sm}^2)$$ *(mass velocity of hot fluid)*

and, $$Re_h := \frac{G_h \cdot Dh_i}{\rho_h \cdot \upsilon_h}$$ *(Remember: $\mu = P \cdot \nu$)*

i.e. $$Re_h = 3.598 \times 10^4$$ *(Reynolds number of hot fluid)*

We have:

$$Nu_h := 0.023 \cdot Re_h^{0.8} \cdot Pr_h^{0.4}$$ *(Dittus–Boelter equation)*

i.e. $$Nu_h = 149.154$$ *(Nusselts number)*

and, $$h_h := \frac{Nu_h \cdot k_h}{Dh_i}$$

i.e. $h_h = 5.212 \times 10^3\ \mathrm{W/(m^2C)}$ *(heat transfer coefficient for hot (inside) fluid)*

Outside heat transfer coefficient:

Equivalent diameter of annulus: $D_{eq} = 4 \times$ (area of cross section/wetted perimeter)

$$Ac_{\text{annulus}} := \pi \cdot \frac{Dc_i^2 - Dh_o^2}{4}$$

i.e. $Ac_{\text{annulus}} = 3.438 \times 10^{-4}\ \mathrm{m^2}$ *(cross-sectional are of annulus)*

and, $P := \pi \cdot (Dc_1 + Dh_o)$

i.e. $P = 0.162\ \mathrm{m}$ *(wetted perimeter of annulus)*

Therefore,

$$D_{eq} := \frac{4 \cdot Ac_{\text{annulus}}}{P}$$

$D_{eq} = 8.5 \times 10^{-3}\ \mathrm{m}$ *(equivalent diameter of annulus)*

Also, $G_c := \frac{m_c}{Ac_{\text{annulus}}}$

i.e. $G_c = 3.232 \times 10^3\ \mathrm{kg/(sm^2)}$ *(mass velocity of cold fluid)*

and, $Re_c := \frac{G_c \cdot D_{eq}}{\nu_c \cdot \rho_c}$

i.e. $Re_c = 4.329 \times 10^3$ *(Reynolds number of cold fluid)*

From Dittus–Boelter's equation

$$Nu_c := 0.023 \cdot Re_c^{0.8} \cdot Pr_c^{0.4}$$

i.e. $Nu_c = 111.152$ *(Nusselts number for cold fluid flow)*

Therefore,

$$h_c := \frac{Nu_c \cdot k_c}{D_{eq}}$$

i.e. $h_c = 1.805 \times 10^3\ \mathrm{W/(m^2C)}$ *(heat transfer coefficient for cold (outside) fluid)*

Overall heat transfer coefficient U:

We have: $U \cdot A = 1/(\text{Total thermal resistance})$

$$R_t := \frac{1}{h_h \cdot (\pi \cdot Dh_i \cdot 1)} + \frac{1}{h_c \cdot (\pi \cdot Dh_o \cdot 1)} + \frac{1}{2 \cdot \pi \cdot k_{cu} \cdot 1} \cdot \ln\left(\frac{Dh_o}{Dh_i}\right)$$ *(Total thermal resistance)*

i.e. $R_t = 0.012\ \mathrm{K/W}$ *(per metre length)*

Therefore,

$$U_i = \frac{1}{R_t \cdot A_i}$$

i.e. $U_i := \frac{1}{R_t \cdot (\pi \cdot Dh_i \cdot 1)}$

i.e. $U_i = 1.471 \times 10^3\ \mathrm{W/(m^2K)}$ *(heat transfer coefficient referred to inside surface area)*

and, $Uo := \frac{1}{R_t \cdot (\pi \cdot Dh_o \cdot 1)}$

i.e. $Uo = 1.287 \times 10^3\ \mathrm{W/(m^2K)}$ *(heat transfer coefficient referred to outside surface area)*

Now, calculate LMTD:

Exit temperature of hot fluid:

$$Th_2 := Th_1 - \frac{Q}{m_h \cdot c_{ph}}$$

i.e. $Th_2 = 52.074°\mathrm{C}$ *(exit temperature of hot fluid)*

Therefore,

$\Delta T_1 := Th_1 - Tc_2$

or, $\Delta T_1 = 55°\mathrm{C}$ *(temperature difference between hot and cold streams at inlet of HX)*

and, $\Delta T_2 := Th_2 - Tc_1$

i.e. $\Delta T_2 = 42.074°\mathrm{C}$ *(temperature difference between hot and cold streams at exit of HX)*

Therefore,

$$\text{LMTD} := \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

i.e. $\text{LMTD} = 48.249°\text{C}$ (*Log Mean Temperature Difference*).

Alternatively:

We can calculate LMTD quickly by using graph of Fig. 12.7 or from Table 12.4: We have: $\Delta T_2/\Delta T_1 = 42.074/55 = 0.765$. From the table, we read against $\Delta T_2/\Delta T_1 = 0.765$, a value of $\text{LMTD}/\Delta T_1 = 0.8775$. Then, $\text{LMTD} = 0.8775 \times 55 = 48.262°\text{C}$.

Length of HX required:

Heat exchange area required:

$$\text{Area} := \frac{Q}{U_i\,\text{LMTD}}$$

i.e. $\text{Area} = 0.295\ \text{m}^2$ (*heat exchange area required*)

Therefore,

$$L := \frac{\text{Area}}{\pi \cdot Dh_i}$$

i.e. $L = 4.993$ m ***(length of HX, metres.)***

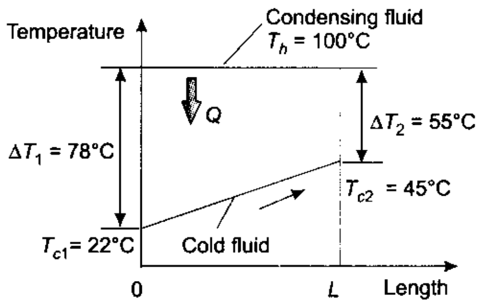


FIGURE Example 12.5 Heat exchanger with one of the fluids condensing ($Ch \Rightarrow \infty$)

Example 12.5. In a shell-and-tube heat exchanger, tubes are 4 m long, 3.1 cm OD, 2.7 cm ID. Water is heated from 22°C to 45°C by condensing steam at 100°C on the outside of tubes. Water flow rate through the tubes is 10 kg/s. Heat transfer coefficient on steam side is 5500 W/(m²K) and on waterside, 850 W/(m²K). Neglecting all other resistances, find the number of tubes.

Solution.

Data:

$Tc_1 := 22°\text{C}$ $\quad Tc_2 := 45°\text{C}$ $\quad T_h := 100°\text{C}$

$h_i := 850\ \text{W/(m}^2\text{K)}$ $\quad h_o := 5500\ \text{W/(m}^2\text{K)}$

$d_i := 0.027$ m $\quad d_o := 0.03$ m $\quad L := 4$ m

$m := 10$ kg/s $\quad c_p := 4170$ J/(kgK)

Overall heat transfer coefficient:

We have: $$U_o \cdot A_o = \frac{1}{\Sigma R_{th}}$$

i.e.
$$U_o \cdot A_o = \frac{1}{\dfrac{1}{h_o \cdot A_o} + \dfrac{1}{h_i \cdot A_i}} = \frac{1}{\dfrac{1}{h_o \cdot \pi \cdot d_o \cdot L} + \dfrac{1}{h_i \cdot \pi \cdot d_i \cdot L}}$$

Since $A_i = \pi \cdot d_i \cdot L$ and $A_o = \pi \cdot d_o \cdot L$

i.e.
$$U_o := \frac{1}{\left(\dfrac{1}{h_o} + \dfrac{d_o}{h_i \cdot d_i}\right)}$$

i.e. $U_o = 671.588\ \text{W/(m}^2\text{K)}$ (*overall heat transfer coefficient referred to outside area*)

To calculate LMTD:

Now, $\Delta T_1 := T_h - Tc_1$

i.e. $\Delta T_1 = 78°\text{C}$ (*temperature difference at inlet*)

and, $\Delta T_2 := T_h - Tc_2$

i.e. $\Delta T_2 = 55°\text{C}$ (*temperature difference at exit*)

Also,
$$\text{LMTD} := \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

i.e. $\text{LMTD} = 65.832°\text{C}$ (*Log Mean Temperature Difference.*)

Alternatively:

We can calculate LMTD quickly by using graph of Fig. 12.7 or from Table 12.4. We have: $\Delta T_2/\Delta T_1 = 55/78 = 0.705$. From the table, we read against $\Delta T_2/\Delta T_1 = 0.705$, value of LMTD/$\Delta T_1$ = 0.844. Then, LMTD = 0.844 × 78 = 65.832°C....same value as obtained above by calculation.

Heat transfer:

$$Q_{total} := m \cdot c_p \cdot (Tc_2 - Tc_1) \text{ W}$$
$$Q_{total} = 9.591 \times 10^5 \text{ W}$$
$$A := \pi \cdot d_o \cdot L;\ A = 0.377$$ *(outside area of each tube, m²)*

Number of tubes required:

$$A_{total} := \frac{Q_{total}}{U_o \cdot \text{LMTD}}$$ *(Total heat transfer area required m²)*

i.e. $$A_{total} = 21.693$$ *(Total heat transfer area required m²)*

Therefore, $$N := \frac{A_{total}}{A}$$

i.e. $$N = 57.543$$ *(No. of tubes required)*

i.e. Number of tubes required is, say, 58.

Example 12.6. In a double pipe counter-flow heat exchanger, 10,000 kg/h of oil (C_p = 2.095 kJ/kgK) is cooled from 80°C to 50°C by 8000 kg/h of water entering at 25°C. Determine the area of heat exchanger for an overall U = 300 W/(m²K). Take C_p for water as 4.18 kJ/kgK.
(M.U. 1997)

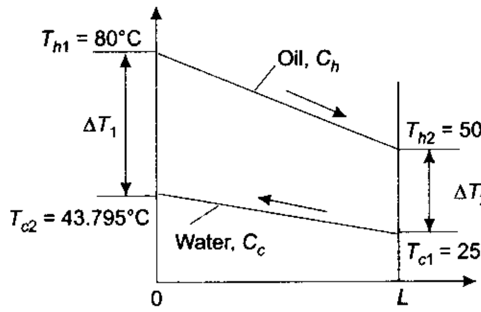


FIGURE Example 12.6 Counter-flow heat exchanger

Solution.

Data:

$$m_h := \frac{10000}{3600} \text{ i.e. } m_h = 2.778 \text{ kg/s}$$

$$C_{ph} := 2095 \text{ J/KgK} \qquad m_c := \frac{8000}{3600}$$

i.e. $m_c = 2.222$ kg/s $\quad C_{pc} := 4180$ J/kgK
$U := 300$ W/(m²K) $\quad T_{h1} := 80$°C
$T_{h2} := 50$°C $\quad T_{c1} := 25$°C

Therefore: $$T_{c2} := Tc_1 + \frac{m_h \cdot C_{ph} \cdot (T_{h1} - T_{h2})}{m_c \cdot C_{pc}}$$ *(from heat balance)*

i.e. $$T_{c2} = 43.795\text{°C}$$ *(exit temperature of cold fluid (water))*

Total heat transfer:

$$Q := m_h \cdot C_{ph} \cdot (T_{h1} - T_{h2})$$

i.e. $$Q = 1.746 \times 10^5 \text{ W}$$ *(total heat transfer)*

To calculate LMTD:

We have:

$$\Delta T_1 := T_{h1} - T_{c2}$$

i.e. $$\Delta T_1 = 36.205\text{°C}$$ *(temperature difference at the inlet of HX)*

and, $$\Delta T_2 := T_{h2} - T_{c1}$$

i.e. $$\Delta T_2 = 25\text{°C}$$ *(temperature difference at the exit of HX)*

Therefore,

$$\text{LMTD} := \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

i.e. $$\text{LMTD} = 30.258\text{°C}$$ *(Log Mean Temperature Difference.)*

Alternatively:

We can calculate LMTD quickly by using graph of Fig. 12.7 or from Table 12.4. We have: $\Delta T_2/\Delta T_1 = 25/36.205 = 0.691$. From the table, we read against $\Delta T_2/\Delta T_1 = 0.69$, a value of LMTD/$\Delta T_1$ = 0.835. Then, LMTD = 0.835 × 36.205 = 30.231°C....almost the same value as obtained above by calculation.

Area required:

$$A := \frac{Q}{U \cdot \text{LMTD}}$$

i.e. $$A = 19.233 \text{ m}^2$$ *(area required.)*

12.5 Correction Factors for Multi-pass and Cross-flow Heat Exchangers

LMTD relations derived above are applicable to parallel flow and counter-flow heat exchangers only. But, in practice, cross-flow heat exchangers (e.g. automobile radiators) and shell-and-tube heat exchangers, with more than one pass in shell side and/or tube side, are also used. In such cases, the flow situation is complex and the analytic relations for mean temperature difference are very complicated. Then, first, LMTD is calculated as if for a counter-flow heat exchanger with the inlet and exit temperatures for the two fluids as per the actual data, and next, a 'correction factor (F)' is applied to the calculated LMTD to get the mean temperature difference between the fluids. Now, heat transfer rate is calculated as:

$$Q = U \cdot A \cdot (F \cdot \text{LMTD}) \text{ W} \qquad \text{...(12.39)}$$

where, A is the area of heat transfer, U is the overall heat transfer coefficient referred to that area, and F is the correction factor. **Note again that LMTD is calculated as if for a counter-flow heat exchanger, taking the inlet and exit temperatures of the two fluids the same as for the actual heat exchanger.**

Values of correction factor (F) for a few selected heat exchangers are given in graphical representation in Fig. 12.9. F varies from 0 to 1. In these graphs, correction factor F is plotted as function of two parameters, i.e. P and R, defined as:

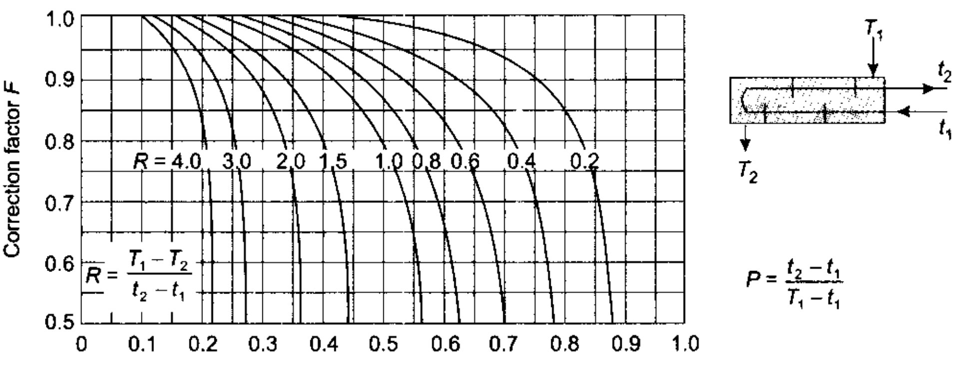


FIGURE 12.9(a) One shell pass and 2,4,6, etc. (any multiple of 2), tube passes

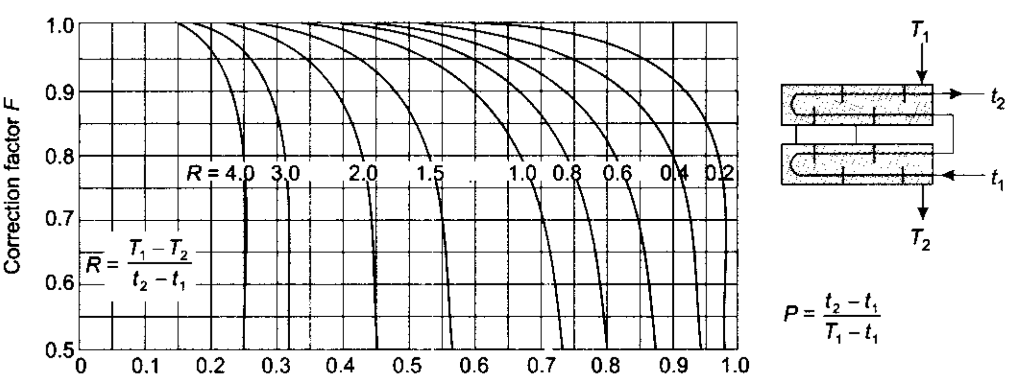


FIGURE 12.9(b) Two shell passes and 4,8,12, etc. (any multiple of 4), tube passes

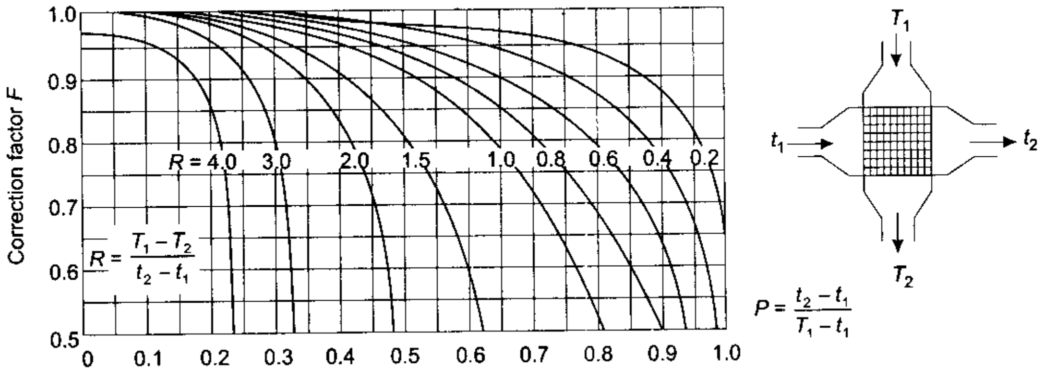


FIGURE 12.9(c) Single pass cross-flow with both fluids un-mixed

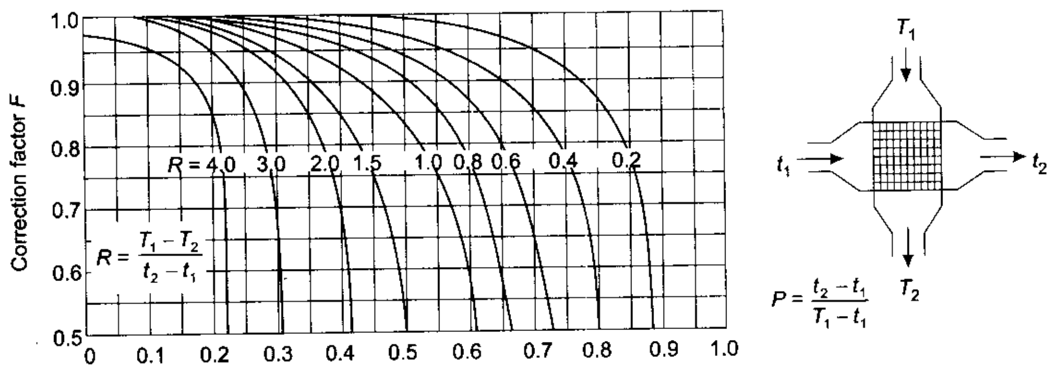


FIGURE 12.9(d) Single pass cross-flow with one fluid mixed and the other un-mixed

$$P = \frac{t_2 - t_1}{T_1 - t_1} \quad \text{...(12.40)}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{C_{tube\_side}}{C_{shell\_side}} \quad \text{...(12.41)}$$

where, C is the capacity rate = $m.C_p$.

Also, for a shell-and-tube heat exchanger, T and t represent the temperatures of fluids flowing through the shell and tube sides, respectively. And, subscripts 1 and 2 refer to the inlet and exit, respectively. It makes no difference whether hot or cold fluid flows through the shell or the tube. Values of P vary from 0 to 1 and it is equal to the ratio of the temperature change of the tube side fluid to the maximum temperature difference between the two fluids; thus, P represents the thermal effectiveness of the tube side fluid; values of R vary from 0 to ∞. When $R = 0$, it means that the fluid on the shell side is undergoing a phase change (i.e. boiling or condensation, which occurs at a practically constant temperature, T_{sat}), and when $R = \infty$, the tube side fluid is undergoing a phase change. Observe from the graphs that, when $R = 0$ or ∞, the correction factor F is equal to 1. Therefore, for a condenser or boiler, $F = 1$, irrespective of the configuration of the heat exchanger.

Note: To apply the correction factor F from these graphs, it is necessary that the end temperatures of both the fluids must be known.

Example 12.7. A one shell pass, two tube pass heat exchanger, with flow arrangement similar to that shown in Fig. 12.9 (a), has water flowing through the tubes and engine oil flowing on the shell side. Water flow rate is 1.2 kg/s and its temperatures at inlet and exit are 25°C and 75°C, respectively. Engine oil enters at 110°C and leaves at 75°C. Overall U = 300 W/(m^{2}K). Take C_p for water as 4.18 kJ/(kgK) and calculate the heat transfer area required.

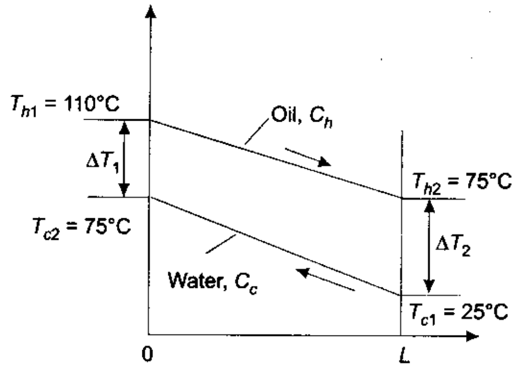


FIGURE Example 12.7 Counter-flow heat exchanger

Solution.

Data:

$m_c := 1.2$ kg/s $\quad C_{pc} := 4180$ J/kgK

$U := 300$ W/(m^{2}K) $\quad T_1 := 110$°C (hot fluid, inlet)

$T_2 := 75$°C (hot fluid, exit)

$t_1 := 25$°C (cold fluid, inlet)

$t_2 := 75$°C (cold fluid, exit)

Therefore, total heat load:

$$Q := m_c \cdot C_{pc} \cdot (t_2 - t_1) \text{ W}$$

i.e. $$Q = 2.508 \times 10^5 \text{ W}$$

Since this is a multi-pass HX, LMTD must be calculated as for a counter-flow HX, and, then a correction factor applied from Fig. 12.9 (a):

To calculate LMTD:

$$\Delta T_1 := T_1 - t_2$$

i.e. $$\Delta T_1 = 35°\text{C}$$

and, $$\Delta T_2 := T_2 - t_1$$

i.e. $$\Delta T_2 = 50°\text{C}$$

Therefore,

$$\text{LMTD} := \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

i.e. $$\text{LMTD} = 42.055°\text{C}$$ *(Log Mean Temperature Difference)*

Correction factor, F:

We have:

$$P := \frac{t_2 - t_1}{T_1 - t_1}$$

i.e. $$P = 0.588$$

and, $$R := \frac{T_1 - T_2}{t_2 - t_1}$$

i.e. $$R = 0.7$$

Then, from Fig. 12.9 (a):

$$F = 0.8$$ *(correction factor)*

And, the corrected temperature difference becomes:

$$\Delta T := 0.8 \cdot \text{LMTD}$$

i.e. $$\Delta T = 33.644°\text{C}$$ *(actual mean temperature)*

Therefore, heat transfer area:

$$A := \frac{Q}{U \cdot \Delta T}$$

i.e. $$A = 24.848 \text{ m}^2$$ ***(heat transfer area required.)***

Example 12.8. In a shell and tube HX, 50 kg/min of furnace oil is heated from 10 to 90°C. Steam at 120°C flows through the shell and oil flows inside the tube. Tube size: 1.65 cm ID and 1.9 cm OD. Heat transfer coefficient on oil and steam sides are: 85 and 7420 W/(m^{2}K), respectively. Find the number of passes and number of tubes in each pass if the length of each tube is limited to 2.85 m. Velocity of oil is limited to 5 cm/s. Density and specific heat of oil are 900 kg/m^3 and 1970 J/(kg.K), respectively. (M.U. 1994)

Solution.

Data:

$T_{c1} := 10$°C $\quad T_{c2} := 90$°C $\quad T_h := 120$°C $\quad d_i := 0.0165$ m $\quad d_o := 0.019$ m $\quad L := 2.85$ m $\quad h_{oil} := 85$ W/(m^{2}K)

$h_{steam} := 7420$ W/(m^{2}K) $\quad V := 0.05$ m/s $\quad \rho_{oil} := 900$ kg/m^3 $\quad Cp_{oil} := 1970$ J/(kgK)

$m_{oil} := \frac{50}{60}$ i.e. $m_{oil} = 0.833$ kg/s

Total heat transferred:

$Q := m_{oil} \cdot Cp_{oil} \cdot (T_{c2} - T_{c1})$

i.e. $Q = 1.313 \times 10^5$ W *(total heat transferred)*

Number of tubes required:

Total cross-sectional area for flow:

$A_{cs} := \frac{m_{oil}}{V \cdot \rho_{oil}}$ i.e. $A_{cs} = 0.019$ m^2

Cross-sectional area of each tube:

$A := \pi \cdot \frac{d_i^2}{4}$ $A = 2.138 \times 10^{-4}$ m^2

Therefore,

$N := \frac{A_{cs}}{A}$ $N = 86.606$ *(number of tubes, say 87 from velocity consideration)*

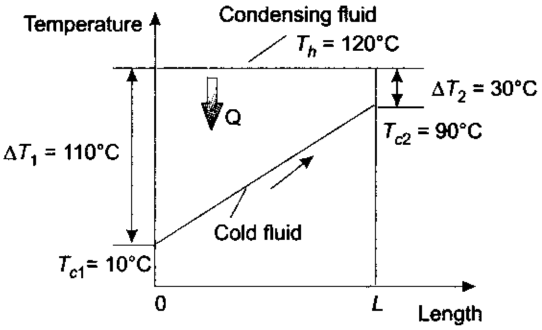


FIGURE Example 12.8 Heat exchanger with one of the fluids condensing ($C_h \Rightarrow \infty$)

i.e. $N = 87$

Overall U, based on outer surface area:

Total thermal resistance:

$$R_t := \frac{1}{h_{oil} \cdot \pi \cdot d_i \cdot 1} + \frac{1}{h_{steam} \cdot \pi \cdot d_o \cdot 1}$$

i.e. $R_t = 0.229$ K/W

Now, $A_o := \pi \cdot do \cdot 1$

i.e. $A_o = 0.06$ m^2 *(outside surface area of tube/metre length)*

Then,

$$U_o := \frac{1}{A_o \cdot R_t}$$

i.e. $U_o = 73.089$ W/(m^{2}K) *(overall heat transfer coefficient referred to outside area of tube)*

and,

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{110 - 30}{\ln\left(\frac{110}{30}\right)}$$

i.e. LMTD = 61.572°C *(Log Mean Temperature Difference)*

Therefore, heat transfer area required:

$$A_{ht} := \frac{Q}{U_o \cdot \text{LMTD}}$$

i.e. $A_{ht} = 29.184$ m^2 *(total heat transfer area required)*

Now, $A_{tube} = \pi \cdot do \cdot 1$

i.e. $A_{tube} = 0.06$ m^2 *(heat transfer area per metre length)*

Therefore,

$$\text{Length} := \frac{A_{ht}}{N \cdot A_{tube}}$$

Length = 5.62 m *(length of tube required)*

But, length is limited to 2.85 m. So, use 2 tube passes.

Then, it becomes a shell-and-tube HX with two tube passes. So, it appears at first sight that correction factor (F) has to be obtained from Fig. 12.9; but, observe that one of the fluids is condensing. So, $F = 1$, irrespective of HX configuration.

i.e. $F = 1$

Therefore, A_{ht} remains same.

Then,

$$\text{Length} = \frac{A_{ht}}{2 \cdot N \cdot A_{tube}}$$

i.e. Length = 2.81 m ***(this is less than 2.85 m, So, OK.)***

12.6 The Effectiveness–NTU Method for Heat Exchanger Analysis

LMTD can readily be determined when all the four end temperatures are either given, or can easily be calculated. Then, the area required, A (i.e. the size of the HX) is easily found out by applying the equation: $Q = U.A.$(LMTD). In other words, LMTD method is very convenient to use for *sizing* problems, when all the end temperatures are known. However, there are certain problems where only the inlet temperatures of both the fluids are specified, along with the flow rates and the overall heat transfer coefficients, and the heat transfer rate and the exit temperatures of the fluids are to be calculated. Solution of such *rating* problems by the LMTD method would require tedious iterations. However, the Effectiveness–NTU method, developed by Kays and London in 1955, overcomes this problem and makes the solution straight forward. Effectiveness–NTU method is also useful in solving heat exchanger problems, where off-design conditions exist; i.e. for example, the heat exchanger might have been designed for some particular flow rates of fluids; now, to find out what happens to the performance if flow rate of one of the fluids is reduced to, say, 75 % of the design flow rate, and so on.

The effectiveness-NTU method is not an altogether new method; fundamental equations are the same as used in the LMTD method, but the different variables are arranged rather differently.

Before we develop the Effectiveness–NTU relations for different types of heat exchangers, let us define a few quantities:

Effectiveness of a heat exchanger (ε):

$$\varepsilon = \frac{Q}{Q_{max}} \quad ...(12.42)$$

where,

Q = actual heat transferred in the heat exhanger

Q_{max} = maximum possible heat transfer in the heat exchanger

Now, actual heat transfer rate in a heat exchanger is given by:

$$Q = m_h \cdot C_{ph} \cdot (T_{h1} - T_{h2}) = C_h \cdot (T_{h1} - T_{h2})$$

and,

$$Q = m_c \cdot C_{pc} \cdot (T_{c2} - T_{c1}) = C_c \cdot (T_{c2} - T_{c1})$$

where, C_h = capacity rate of the hot fluid, and

C_c = capacity rate of the cold fluid

Now, C_h may be equal to C_c or less than C_c or greater than C_c.

If $C_h < C_c$, we designate C_h as C_{min};

Instead, if $C_h > C_c$, we designate C_c as C_{min}.

And, in each case, capacity rate of the other fluid is designated as C_{max}.

Capacity Ratio (C):

Capacity ratio is defined as:

$$C = \frac{C_{min}}{C_{max}} \quad ...(12.43)$$

Number of Transfer Units (NTU):

Number of Transfer Units (which is a dimensionless number), is defined as:

$$\text{NTU} = \frac{U \cdot A}{C_{min}} \quad ...(12.44)$$

where, U is the overall heat transfer coefficient and A is the corresponding heat transfer area. For given value of A and flow conditions, NTU is a measure of the area (i.e. size) of the heat exchanger. Larger the NTU, larger the size of the heat exchanger.

Maximum possible heat transfer in a heat exchanger (Q_{max}):

Now, consider a heat exchanger where the hot fluid is cooled from a temperature of T_{h1} to T_{h2} and the cold fluid heated from T_{c1} to T_{c2}. So, the maximum temperature differential in the heat exchanger is $(T_{h1} - T_{c1})$. Now, if the heat exchanger had an infinite area, the hot fluid will be cooled from T_{h1} to T_{c1} or the cold fluid may be heated from T_{c1} to T_{h1}. However, which fluid will experience the maximum temperature differential $(T_{h1} - T_{c1})$ will depend upon which fluid has the minimum capacity rate.

If hot fluid has the minimum capacity rate, we can write:

$$Q_{max} = C_h \cdot (T_{h1} - T_{c1}) \qquad \textit{(if } C_h \textit{ is minimum capacity rate, } C_{min})$$

Instead, if cold fluid has the minimum capacity rate, we write:

$$Q_{max} = C_c \cdot (T_{h1} - T_{c1}) \qquad \textit{(if } C_c \textit{ is minimum capacity rate, } C_{min}.)$$

Or, more generally, we write:

$$Q_{max} = C_{min} \cdot (T_{h1} - T_{c1}) \qquad ...(12.45)$$

These situations are represented graphically in Fig. 12.9:

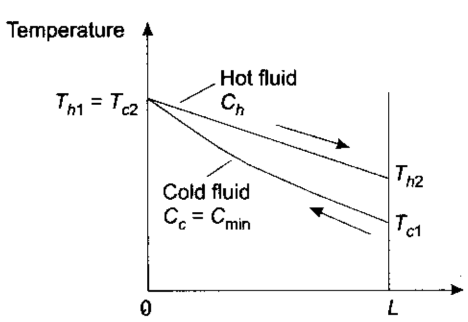


FIGURE 12.9(a) Cold fluid has minimum capacity rate

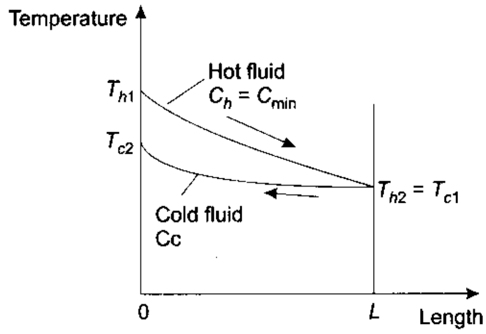


FIGURE 12.9(b) Hot fluid has minimum capacity rate

Therefore, we can write for effectiveness:

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{C_h \cdot (T_{h1} - T_{h2})}{C_{min} \cdot (T_{h1} - T_{c1})} = \frac{C_c \cdot (T_{c2} - T_{c1})}{C_{min} \cdot (T_{h1} - T_{c1})} \qquad ...(12.46)$$

Now, if hot fluid is the 'minimum fluid' (i.e. $C_h < C_c$), we get from Eq. 12.46:

$$\varepsilon = \frac{(T_{h1} - T_{h2})}{(T_{h1} - T_{c1})} \qquad \textit{(for } C_h < C_c \textit{...(12.47a).)}$$

And, if cold fluid is the 'minimum fluid' (i.e. $C_c < C_h$), we get from Eq. 12.46:

$$\varepsilon = \frac{(T_{c2} - T_{c1})}{(T_{h1} - T_{c1})} \qquad \textit{(for } C_c < C_h \textit{...(12.47b))}$$

i.e. by suitably choosing the fluid, the effectiveness of a heat exchanger can be expressed as a ratio of temperatures (or, as a temperature effectiveness).

If $C_h = C_c$, obviously, both the fluids will experience the maximum possible temperature differential, if the heat exchanger had an infinite area.

Now, for any heat exchanger, effectiveness can be expressed as a function of the NTU and capacity ratio, C_{min}/C_{max}, i.e.

$$\varepsilon = f\left(\mathrm{NTU}, \frac{C_{min}}{C_{max}}\right) \qquad ...(12.47c)$$

We shall derive below ε-NTU relation for a parallel flow HX.

12.6.1 *Effectiveness–NTU Relation for a Parallel-flow Heat Exchanger*

Consider the parallel-flow heat exchanger shown in Fig. 12.5. Assumptions for this derivation remain the same as for the LMTD method.

Continuing from Eq. 12.21:

$$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right) = -U \cdot A \cdot \left(\frac{1}{m_h \cdot C_{ph}} + \frac{1}{m_c \cdot C_{pc}}\right) \qquad ...(12.21)$$

Now, out of the two fluids, one is the 'minimum' fluid and the other is the 'maximum' fluid. Whichever may be the minimum fluid, we can write Eq. 12.21 as:

$$\ln\left(\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}}\right) = \frac{-U\cdot A}{C_{min}}\cdot\left(1+\frac{C_{min}}{C_{max}}\right) \quad ...(12.48)$$

i.e.
$$\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}} = \exp\left[-\text{NTU}\cdot\left(1+\frac{C_{min}}{C_{max}}\right)\right] \quad ...(12.49)$$

Now, substituting for T_{h2} and T_{c2} from Eq. 12.46, we get:

$$\frac{\left[T_{h1}-\varepsilon\cdot\frac{C_{min}}{C_h}\cdot(T_{h1}-T_{c1})\right]-\left[\varepsilon\cdot\frac{C_{min}}{C_c}\cdot(T_{h1}-T_{c1})+T_{c1}\right]}{T_{h1}-T_{c1}} = \exp\left[-\text{NTU}\cdot\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

i.e.
$$\frac{(T_{h1}-T_{c1})-\varepsilon\cdot C_{min}\cdot(T_{h1}-T_{c1})\cdot\left(\frac{1}{C_h}+\frac{1}{C_c}\right)}{T_{h1}-T_{c1}} = \exp\left[-\text{NTU}\cdot\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

i.e.
$$1-\varepsilon\cdot C_{min}\cdot\left(\frac{1}{C_h}+\frac{1}{C_c}\right) = \exp\left[-\text{NTU}\cdot\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

Now, assuming $C_h > C_c$, i.e. cold fluid as the 'minimum fluid', we have: $C_{min} = C_c$ and $C_{max} = C_h$. Therefore,

$$1-\varepsilon\cdot\left(1+\frac{C_{min}}{C_{max}}\right) = \exp\left[-\text{NTU}\cdot\left(1+\frac{C_{min}}{C_{max}}\right)\right]$$

i.e.
$$\varepsilon = \frac{1-\exp\left[-\text{NTU}\cdot\left(1+\frac{C_{min}}{C_{max}}\right)\right]}{1+\frac{C_{min}}{C_{max}}} \quad ...(12.50)$$

Eq. 12.50 is the desired expression for effectiveness of a parallel flow heat exchanger.

Note that the same result would be obtained, if we assume the hot fluid as the 'minimum' fluid.

Eq. 12.50 is concisely expressed as:

$$\varepsilon = \frac{1-\exp(-N\cdot(1+C))}{1+C} \quad ...(12.51)$$

where, N = NTU and, $$C = \frac{C_{min}}{C_{max}}$$

Special cases:

(i) **For a condenser or boiler** i.e. one of the fluids undergoes a phase change. Therefore, $C_{max} \to \infty$ i.e. Capacity ratio, $C = 0$. Then effectiveness relation (for all heat exchangers) reduces to:

$$\varepsilon = 1-\exp(-\text{NTU}) \quad ...(12.52)$$

(ii) **When $C = 1$, i.e. $C_{min} = C_{max}$** This is the case of a typical, gas turbine regenerator. In this case,

$$\varepsilon = \frac{1-\exp(-2\cdot\text{NTU})}{2} \quad \textit{(for C = 1, parallel flow HX...(12.53))}$$

12.6.2 Effectiveness–NTU Relation for a Counter-flow Heat Exchanger

Again, consider the counter-flow heat exchanger shown in Fig. 12.6. Assumptions for this derivation remain the same as for the LMTD method.

Continuing from Eq. 12.31:

$$\ln\left(\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}}\right) = -U\cdot A\cdot\left(\frac{1}{m_h\cdot C_{ph}} - \frac{1}{m_c\cdot C_{pc}}\right) \quad ...(12.31)$$

This can be written as:

$$\ln\left(\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}}\right) = -U\cdot A\cdot\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

Assuming hot fluid as the 'minimum' fluid,

$$C_{min} = C_h \quad \text{and,} \; C_{max} = C_c$$

we have:

$$\ln\left(\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}}\right) = -U\cdot A\cdot\left(\frac{1}{C_{min}} - \frac{1}{C_{max}}\right)$$

i.e.

$$\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}} = \exp\left[-U\cdot A\cdot\left(\frac{1}{C_{min}} - \frac{1}{C_{max}}\right)\right]$$

i.e.

$$\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}} = \exp\left[\frac{-U\cdot A}{C_{min}}\cdot\left(1 - \frac{C_{min}}{C_{max}}\right)\right]$$

i.e.

$$\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}} = \exp\left[-\text{NTU}\cdot\left(1 - \frac{C_{min}}{C_{max}}\right)\right] \quad ...(12.54)$$

Now, substituting for T_{h2} and T_{c2} from Eq. 12.46, we get:

$$\frac{\left[T_{h1} - \varepsilon\cdot\frac{C_{min}}{C_h}\cdot(T_{h1}-T_{c1}) - T_{c1}\right]}{\left[T_{h1} - \varepsilon\cdot\frac{C_{min}}{C_c}\cdot(T_{h1}-T_{c1}) - T_{c1}\right]} = \exp\left[-\text{NTU}\cdot\left(1 - \frac{C_{min}}{C_{max}}\right)\right] \quad ...(12.55)$$

Now, put $C_{min} = C_h$, $C = C_{min}/C_{max}$, and N = NTU, in Eq. 12.55:

$$\frac{(T_{h1}-T_{c1})\cdot(1-\varepsilon)}{(T_{h1}-T_{c1})\cdot(1-C\cdot\varepsilon)} = \exp(-N\cdot(1-C))$$

i.e.

$$\frac{1-\varepsilon}{1-C\cdot\varepsilon} = \exp(-N\cdot(1-C))$$

i.e.

$$1-\varepsilon = \exp(-N\cdot(1-C)) - C\cdot\varepsilon\cdot\exp(-N\cdot(1-C))$$

i.e.

$$\varepsilon\cdot(1 - C\cdot\exp(-N\cdot(1-C))) = 1 - \exp\cdot(-N\cdot(1-C))$$

or,

$$\varepsilon = \frac{1-\exp(-N\cdot(1-C))}{(1-C\cdot\exp(-N\cdot(1-C)))} \quad ...(12.56)$$

Instead of assuming that the hot fluid is the minimum fluid, if we assume that the cold fluid is the 'minimum' fluid, then also the same relation (namely, Eq. 12.56), will result.

Eq. 12.56 is the desired expression for the effectiveness of the counter-flow heat exchanger.

Special cases:

(i) **For a condenser or boiler** i.e. one of the fluids undergoes a phase change. Therefore, $C_{max} \to \infty$. i.e. Capacity ratio, $C = 0$. Then effectiveness relation (for all heat exchangers) reduces to:

$$\varepsilon = 1 - \exp(-\text{NTU}) \quad ...(12.57)$$

(ii) **When $C = 1$, i.e. $C_{min} = C_{max}$** This is the case of a typical, gas turbine regenerator. In this case, relation for ε reduces to the indeterminate form, 0/0. Then, apply the L'Hospital's rule to evaluate ε. i.e. differentiate the numerator and denominator w.r.t. C and taking the limit $C \to 1$, we get:

$$\varepsilon = \frac{NTU}{1 + NTU} \qquad \text{-for } C = 1., \text{ counter-flow HX...(12.58))}$$

Effectiveness–NTU relations and the corresponding graphical representations for several types of heat exchangers are given by Kays and London.

Table 12.5 gives the Effectiveness relations for a few types of heat exchangers; and Table 12.6 gives the NTU relations:

TABLE 12.5 Effectiveness relations for heat exchangers
[N = NTU = $U.A/C_{min}$, $C = C_{min}/C_{max}$]

Flow geometry	Relation
Double pipe: parallel-flow	$\varepsilon = \dfrac{1-\exp(-N\cdot(1+C))}{1+C}$
Double pipe: counter-flow	$\varepsilon = \dfrac{1-\exp(-N\cdot(1-C))}{(1-C\cdot\exp(-N\cdot(1-C)))}$
Counter-flow, $C = 1$	$\varepsilon = \dfrac{N}{1+N}$
Cross-flow: (single pass) both fluids un-mixed	$\varepsilon = 1 - \exp\left(\dfrac{\exp(-N\cdot C\cdot n)-1}{C\cdot n}\right)$ where, $n = N^{-0.22}$
Cross-flow: (single pass) both fluids mixed	$\varepsilon = \left(\dfrac{1}{1-\exp(-N)} + \dfrac{C}{1-\exp(-N\cdot C)} - \dfrac{1}{N}\right)^{-1}$
Cross-flow: (single pass) C_{max} mixed, C_{min} un-mixed	$\varepsilon = \dfrac{1}{C}\cdot[1 - \exp[-C\cdot(1 - e^{-N})]]$
Cross-flow: (single pass) C_{max} un-mixed, C_{min} mixed	$\varepsilon = 1 - \exp\left[-\dfrac{1}{C}\cdot(1-\exp(-N\cdot C))\right]$
Shell and tube:	
One shell pass, 2, 4, 6 tube passes	$\varepsilon = 2\cdot\left[1+C+(1+C^2)^{\frac{1}{2}}\cdot\dfrac{1+\exp\left[-N\cdot(1+C^2)^{\frac{1}{2}}\right]}{1-\exp\left[-N\cdot(1+C^2)^{\frac{1}{2}}\right]}\right]^{-1}$
Multiple shell passes, 2n, 4n, 6n tube passes (ε_p = effectiveness of each shell pass, n = number. of shell passes)	$\varepsilon = \dfrac{\left[\dfrac{(1-\varepsilon_p\cdot C)}{(1-\varepsilon_p)}\right]^n - 1}{\left[\dfrac{(1-\varepsilon_p\cdot C)}{(1-\varepsilon_p)}\right]^n - C}$
Special case for $C = 1$	$\varepsilon = \dfrac{n\cdot\varepsilon_p}{1+(n-1)\cdot\varepsilon_p}$
All exchangers, with $C = 0$ (Condensers and Evaporators)	$\varepsilon = 1 - e^{-N}$

TABLE 12.6 NTU relations for heat exchangers
[N = NTU = UA/C_{min}, C = C_{min}/C_{max}, ε = effectiveness]

Flow geometry	Relation
Double pipe: parallel-flow	$N = \dfrac{-\ln(1-(1+C)\cdot\varepsilon)}{1+C}$
Double pipe: counter-flow, for C = 1	$N = \dfrac{1}{C-1}\cdot\ln\left(\dfrac{\varepsilon-1}{C\cdot\varepsilon-1}\right)$
Counter-flow, $C \doteq 1$	$N = \dfrac{\varepsilon}{1-\varepsilon}$
Cross-flow: C_{max} mixed, C_{min} un-mixed	$N = -\ln\left(1+\dfrac{1}{C}\cdot\ln(1-C\cdot\varepsilon)\right)$
Cross-flow: C_{max} un-mixed, C_{min} mixed	$N = \dfrac{-1}{C}\cdot\ln(1+C\cdot\ln(1-\varepsilon))$
Shell and tube:	
One shell pass, 2, 4, 6 tube passes	$N = -(1+C^2)^{\frac{-1}{2}}\cdot\ln\left[\dfrac{\frac{2}{\varepsilon}-1-C-(1+C^2)^{\frac{1}{2}}}{\frac{2}{\varepsilon}-1-C+(1+C^2)^{\frac{1}{2}}}\right]$
All exchangers, with C = 0 (Condensers and Evaporators)	$N = -\ln(1-\varepsilon)$

NTU–Effectiveness graphs:

NTU–Effectiveness relations are also represented in graphical form and these are quite instructive. However, it is a bit difficult to read these graphs accurately; so, analytical relations may be used wherever possible.

NTU–Effectiveness relations for parallel-flow and counter-flow heat exchangers are shown graphically in Fig. 12.10 and 12.11, respectively. In these figures, effectiveness values are plotted against NTU for different values of capacity ratio, C.

For convenience and accuracy in reading, effectiveness values for the parallel flow and counter-flow heat exchangers are given in Tabular form, in Table 12.7 and 12.8:

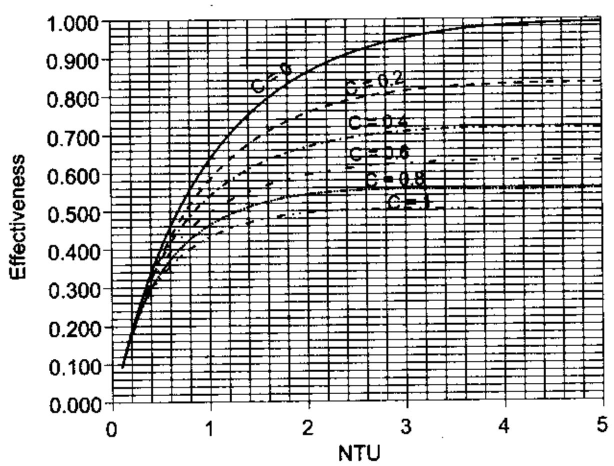


FIGURE 12.10 NTU Vs. effectiveness for parallel-flow heat exchangers

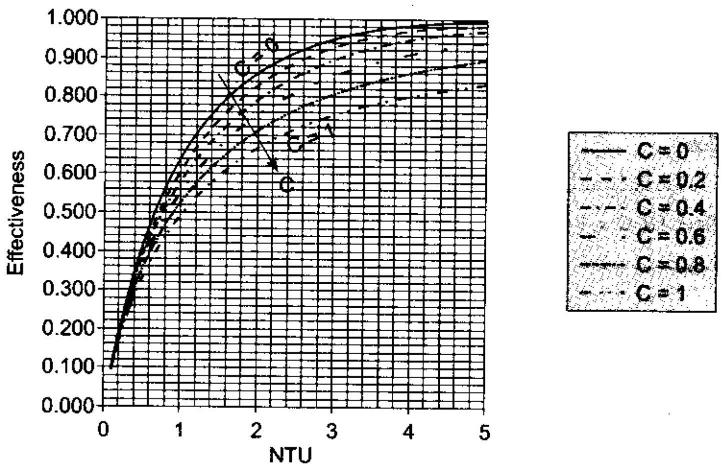


FIGURE 12.11 NTU Vs. effectiveness for counter-flow heat exchangers

NTU–effectiveness graphs for some other types of heat exchangers, are given by Kays and London, and are reproduced below:

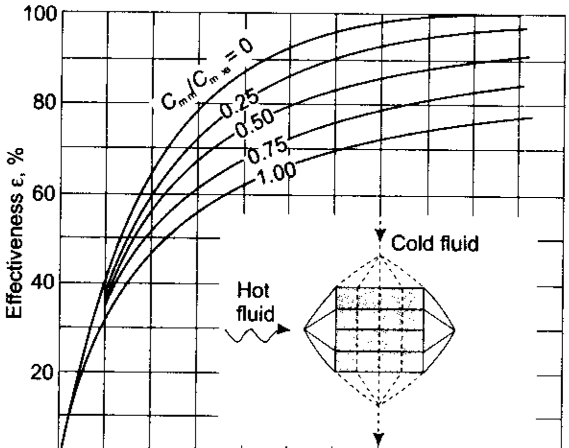


FIGURE 12.12 Cross-flow heat exchanger with both fluids un-mixed

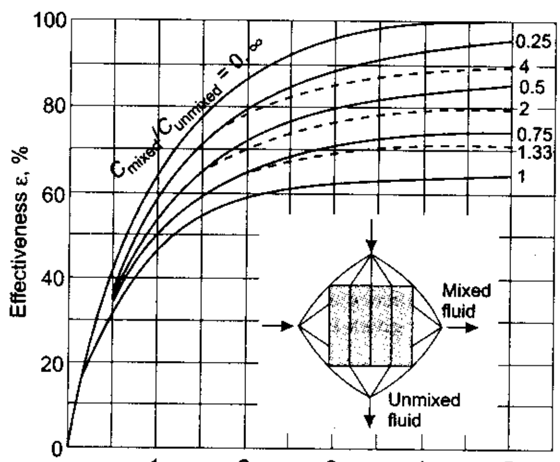


FIGURE 12.13 Cross-flow heat exchanger with one fluid mixed and the other un-mixed

Note: In Fig. 12.13, the dashed lines are for the case of C_{min} un-mixed and C_{max} mixed. And, the solid lines are for the case of C_{min} mixed and C_{max} un-mixed.

From the NTU—Effectiveness graphs, following important points may be observed:

(i) For a given value of capacity ratio, C, the effectiveness increases with NTU. Value of effectiveness varies from 0 to 1.

(ii) Initially, effectiveness increases rather rapidly as NTU increases (up to a value of NTU = about 1.5) and then, slowly for larger values of NTU. Remember that NTU is a measure of the size (i.e. heat exchange area, A) of the heat exchanger; so, we can conclude that increasing the size of the heat exchanger beyond about NTU = 3, cannot be economically justified, since there will not be any corresponding increase in effectiveness.

TABLE 12.7 NTU Vs. effectiveness for parallel-flow HX

NTU	C = 0	C = 0.2	C = 0.4	C = 0.6	C = 0.8	C = 1
0.1	0.095	0.094	0.093	0.092	0.092	0.091
0.2	0.181	0.178	0.174	0.171	0.168	0.165
0.3	0.259	0.252	0.245	0.238	0.232	0.226
0.4	0.330	0.318	0.306	0.295	0.285	0.275
0.5	0.393	0.376	0.360	0.344	0.330	0.316
0.6	0.451	0.428	0.406	0.386	0.367	0.349
0.7	0.503	0.474	0.446	0.421	0.398	0.377
0.8	0.551	0.514	0.481	0.451	0.424	0.399
0.9	0.593	0.550	0.512	0.477	0.446	0.417
1	0.632	0.582	0.538	0.499	0.464	0.432
1.1	0.667	0.611	0.561	0.517	0.479	0.445
1.2	0.699	0.636	0.581	0.533	0.491	0.455
1.3	0.727	0.658	0.599	0.547	0.502	0.463
1.4	0.753	0.678	0.614	0.558	0.511	0.470
1.5	0.777	0.696	0.627	0.568	0.518	0.475
1.6	0.798	0.711	0.638	0.577	0.524	0.480
1.7	0.817	0.725	0.648	0.584	0.530	0.483
1.8	0.835	0.737	0.657	0.590	0.534	0.486
1.9	0.850	0.748	0.664	0.595	0.537	0.489
2	0.865	0.758	0.671	0.600	0.540	0.491
2.1	0.878	0.766	0.677	0.603	0.543	0.493
2.2	0.889	0.774	0.681	0.607	0.545	0.494
2.3	0.900	0.781	0.686	0.609	0.547	0.495
2.4	0.909	0.787	0.689	0.612	0.548	0.496
2.5	0.918	0.792	0.693	0.614	0.549	0.497
2.6	0.926	0.797	0.696	0.615	0.550	0.497
2.7	0.933	0.801	0.698	0.617	0.551	0.498
2.8	0.939	0.804	0.700	0.618	0.552	0.498
2.9	0.945	0.808	0.702	0.619	0.553	0.498
3	0.950	0.811	0.704	0.620	0.553	0.499
3.1	0.955	0.813	0.705	0.621	0.553	0.499
3.2	0.959	0.815	0.706	0.621	0.554	0.499
3.3	0.963	0.817	0.707	0.622	0.554	0.499
3.4	0.967	0.819	0.708	0.622	0.554	0.499
3.5	0.970	0.821	0.709	0.623	0.555	0.500
3.6	0.973	0.822	0.710	0.623	0.555	0.500
3.7	0.975	0.824	0.710	0.623	0.555	0.500
3.8	0.978	0.825	0.711	0.624	0.555	0.500
3.9	0.980	0.826	0.711	0.624	0.555	0.500
4	0.982	0.826	0.712	0.624	0.555	0.500
4.1	0.983	0.827	0.712	0.624	0.555	0.500
4.2	0.985	0.828	0.712	0.624	0.555	0.500
4.3	0.986	0.829	0.713	0.624	0.555	0.500
4.4	0.988	0.829	0.713	0.624	0.555	0.500
4.5	0.989	0.830	0.713	0.625	0.555	0.500
4.6	0.990	0.830	0.713	0.625	0.555	0.500
4.7	0.991	0.830	0.713	0.625	0.555	0.500
4.8	0.992	0.831	0.713	0.625	0.555	0.500
4.9	0.993	0.831	0.714	0.625	0.555	0.500
5	0.993	0.831	0.714	0.625	0.555	0.500

TABLE 12.8 NTU Vs. effectiveness for counter-flow HX

NTU	C = 0	C = 0.2	C = 0.4	C = 0.6	C = 0.8	C = 1
0.1	0.095	0.094	0.093	0.092	0.092	0.091
0.2	0.181	0.178	0.175	0.172	0.169	0.167
0.3	0.259	0.253	0.247	0.242	0.236	0.231
0.4	0.330	0.320	0.311	0.303	0.294	0.286
0.5	0.393	0.381	0.368	0.356	0.345	0.333
0.6	0.451	0.435	0.419	0.404	0.389	0.375
0.7	0.503	0.484	0.465	0.447	0.429	0.412
0.8	0.551	0.528	0.507	0.485	0.465	0.444
0.9	0.593	0.569	0.544	0.520	0.496	0.474
1	0.632	0.605	0.578	0.551	0.525	0.500
1.1	0.667	0.638	0.609	0.580	0.552	0.524
1.2	0.699	0.668	0.637	0.606	0.576	0.545
1.3	0.727	0.696	0.663	0.630	0.598	0.565
1.4	0.753	0.721	0.687	0.652	0.618	0.583
1.5	0.777	0.744	0.709	0.673	0.636	0.600
1.6	0.798	0.764	0.729	0.691	0.653	0.615
1.7	0.817	0.784	0.747	0.709	0.669	0.630
1.8	0.835	0.801	0.764	0.725	0.684	0.643
1.9	0.850	0.817	0.780	0.740	0.698	0.655
2	0.865	0.832	0.795	0.754	0.711	0.667
2.1	0.878	0.845	0.808	0.767	0.723	0.677
2.2	0.889	0.857	0.821	0.779	0.734	0.688
2.3	0.900	0.869	0.832	0.790	0.745	0.697
2.4	0.909	0.879	0.843	0.801	0.755	0.706
2.5	0.918	0.889	0.853	0.811	0.764	0.714
2.6	0.926	0.897	0.862	0.821	0.773	0.722
2.7	0.933	0.906	0.871	0.829	0.782	0.730
2.8	0.939	0.913	0.879	0.838	0.790	0.737
2.9	0.945	0.920	0.887	0.846	0.797	0.744
3	0.950	0.926	0.894	0.853	0.804	0.750
3.1	0.955	0.932	0.900	0.860	0.811	0.756
3.2	0.959	0.937	0.907	0.867	0.818	0.762
3.3	0.963	0.942	0.912	0.873	0.824	0.767
3.4	0.967	0.947	0.918	0.879	0.830	0.773
3.5	0.970	0.951	0.923	0.884	0.835	0.778
3.6	0.973	0.955	0.927	0.890	0.841	0.783
3.7	0.975	0.958	0.932	0.895	0.846	0.787
3.8	0.978	0.961	0.936	0.899	0.851	0.792
3.9	0.980	0.964	0.940	0.904	0.855	0.796
4	0.982	0.967	0.944	0.908	0.860	0.800
4.1	0.983	0.970	0.947	0.912	0.864	0.804
4.2	0.985	0.972	0.950	0.916	0.868	0.808
4.3	0.986	0.974	0.953	0.920	0.872	0.811
4.4	0.988	0.976	0.956	0.923	0.876	0.815
4.5	0.989	0.978	0.959	0.927	0.879	0.818
4.6	0.990	0.980	0.961	0.930	0.883	0.821
4.7	0.991	0.981	0.963	0.933	0.886	0.825
4.8	0.992	0.983	0.966	0.936	0.890	0.828
4.9	0.993	0.984	0.968	0.938	0.893	0.831
5	0.993	0.985	0.970	0.941	0.896	0.833

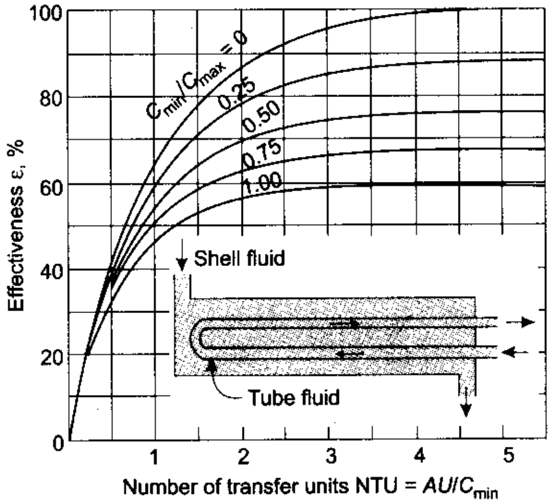


FIGURE 12.14 Shell and tube heat exchanger, with one shell pass and 2, 4, 6 tube passes

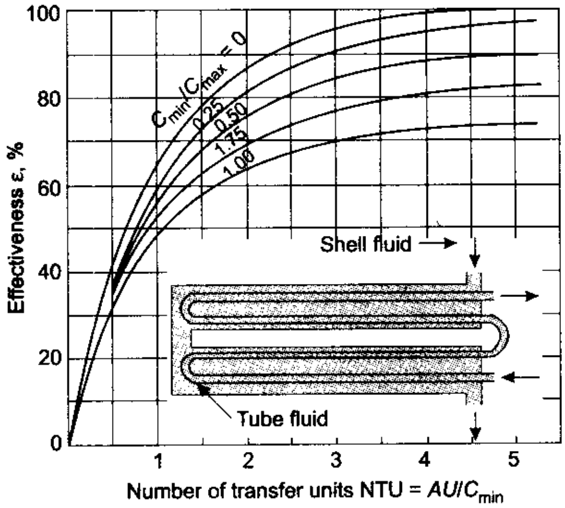


FIGURE 12.15 Shell and tube heat exchanger, with two shell passes and 4, 8, 12 tube passes

(iii) At a given value of NTU, effectiveness is maximum for $C = 0$, (i.e. for a condenser or evaporator), and decreases as C increases.

(iv) For NTU less than about 0.3, effectiveness is independent of capacity ratio, C.

(v) For given NTU and C, a counter-flow heat exchanger has highest effectiveness and a parallel, flow heat exchanger has the lowest effectiveness.

(vi) When $C = 1$ (i.e. capacity rates of both the fluids are equal, as in the case of a typical regenerator), maximum effectiveness of a parallel-flow heat exchanger is 50% only, whereas there is no such limitation for a counter-flow HX. Therefore, for such applications, obviously, the counter-flow arrangement is preferred.

Example 12.9. Consider a heat exchanger for cooling oil which enters at 180°C, and cooling water enters at 25°C. Mass flow rates of oil and water are: 2.5 and 1.2 kg/s, respectively. Area for heat transfer = 16 m^2. Specific heat data for oil and water and overall U are given: $Cp_{oil} = 1900$ J/kgK; $Cp_{water} = 4184$ J/kgK; $U = 285$ W/m^{2}K. Calculate outlet temperatures of oil and water for parallel and counter-flow HX. (M.U. 1995)

Solution. Here, the outlet temperatures of both the fluids are not known. Use of LMTD method would require an iterative solution. i.e. to start with, assume outlet temperature of, say, hot fluid, T_{h2} and calculate the exit temperature of cold fluid, T_{c2} and then, the LMTD; then, calculate the heat transfer rate Q. From Q and capacity rates, recalculate T_{h2}, and compare this value with the initially assumed value; if they do not match, say, within 0.5 deg.C, repeat the iterative cycle.

But, as will be shown below, Effectiveness–NTU method, offers a direct, straightforward solution:

Data:

$m_h := 2.5$ kg/s $\quad m_c := 1.2$ kg/s $\quad T_{h1} := 180°C \quad U := 285$ W/(m^{2}K) $\quad A := 16$ m^2 $\quad T_{c1} := 25°C$

$C_{ph} := 1900$ J/(kgK) $\quad C_{pc} := 4184$ J/(kgK)

Capacity rates:

$$C_h := m_h \cdot C_{ph}$$

i.e.

$$C_h = 4.75 \times 10^3 \text{ W/K}$$

and,

$$C_c := m_c \cdot C_{pc}$$

i.e.

$$C_c = 5.021 \times 10^3 \text{ W/K}$$

Therefore,

$$C_{min} := C_h \text{ W/K} \quad \textit{(minimum capacity rate)}$$

and,

$$C_{max} := C_c \text{ W/K} \quad \textit{(maximum capacity rate)}$$

Therefore, Capacity ratio:

$$C := \frac{C_{min}}{C_{max}}$$

i.e. $C = 0.946$ *(capacity ratio)*

Number of Transfer Units:

$$\text{NTU} := \frac{U \cdot A}{C_{min}}$$

i.e. $\text{NTU} = 0.96$

Case (i): Parallel-flow HX:

For parallel-flow HX, we have the effectiveness relation:

$$\varepsilon = \frac{1 - \exp\left[-\text{NTU}\cdot\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}} \quad ...(12.50)$$

$$\varepsilon := \frac{1 - \exp(-\text{NTU}\cdot(1 + C))}{1 + C}$$

i.e. $\varepsilon = 0.435$ *(effectiveness of parallel-flow HX)*

Then, since hot fluid is the 'minimum' fluid, we have:

$$\varepsilon = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$$

i.e. $T_{h2} := T_{h1} - \varepsilon\cdot(T_{h1} - T_{c1})$

i.e. $T_{h2} = 112.65°\text{C}$ *(exit temperature of hot fluid (oil))*

and, T_{c2} is obtained from heat balance:

$$C_h\cdot(T_{h1} - T_{h2}) = C_c\cdot(T_{c2} - T_{c1}) \quad \textit{(by heat balance)}$$

i.e. $T_{c2} := T_{c1} + \frac{C_h\cdot(T_{h1} - T_{h2})}{C_c}$

i.e. $T_{c2} = 88.718°\text{C}$ ***(exit temperature of cold fluid (water))***

Case (ii): Counter-flow HX:

For counter-flow HX, we have:

$$\varepsilon := \frac{1 - \exp(-\text{NTU}\cdot(1 - C))}{(1 - C\cdot\exp(-\text{NTU}\cdot(1 - C)))} \quad ...(12.56)$$

i.e. $\varepsilon = 0.496$ *(effectiveness of counter-flow HX)*

Then, again, since hot fluid is the 'minimum' fluid, we have:

$$\varepsilon = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$$

i.e. $T_{h2} := T_{h1} - \varepsilon\cdot(T_{h1} - T_{c1})$

i.e. $T_{h2} = 103.074°\text{C}$ ***(exit temperature of hot fluid (oil).)***

And, T_{c2} is obtained from:

$$T_{c2} := T_{c1} + \frac{C_h\cdot(T_{h1} - T_{h2})}{C_c}$$

i.e. $T_{c2} = 97.777°\text{C}$ ***(exit temperature of cold fluid (water).)***

Note: In this problem, it is difficult to read accurately the ε values from the graphs, for the given values of NTU and C. It is suggested that the analytical relations may be used to get accurate results.

(b) In the above Example, suppose that the flow rate of water is increased to 2 kg/s.

Calculate the new outlet temperatures of oil and water for parallel and counter-flow HX. Rest of the data remain the same.

Now, the heat exchanger is operated at an off-design condition, i.e. the water flow is changed from 1.2 kg/s to 2 kg/s. Then, ε-NTU method is convenient to use to find out the exit temperatures of both the fluids.

$m_c := 2\ \text{kg/s}$ *(mass flow rate of cold fluid (water))*

Note that still, hot fluid is the 'minimum' fluid and NTU remains the same, but C changes:

Capacity rates:

$$C_h := m_h\cdot C_{ph}$$

i.e. $C_h = 4.75 \times 10^3$ W/K

and, $C_c := m_c \cdot C_{pc}$

i.e. $C_c = 8.368 \times 10^3$ W/K

Therefore,

$C_{min} := C_h$ W/K *(minimum capacity rate)*

and, $C_{max} := C_c$ W/K *(maximum capacity rate)*

Therefore, Capacity ratio:

$$C := \frac{C_{min}}{C_{max}}$$

i.e. $C = 0.568$ *(capacity ratio)*

and, $$NTU := \frac{U \cdot A}{C_{min}}$$

i.e. NTU = 0.96

Case (i): Parallel-flow HX:

We have the effectiveness relation:

$$\varepsilon := \frac{1 - \exp(-NTU \cdot (1 + C))}{1 + C}$$

i.e. $\varepsilon = 0.496$ *(effectiveness of parallel-flow HX)*

Compare this ε with $\varepsilon = 0.435$ obtained earlier.

Then, since hot fluid is the 'minimum' fluid, we have:

$$\varepsilon = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$$

i.e. $T_{h2} := T_{h1} - \varepsilon \cdot (T_{h1} - T_{c1})$

i.e. $T_{h2} = 103.079°C$ ***(exit temperature of hot fluid (oil).)***

And, T_{c2} is obtained from heat balance:

$C_h = (T_{h1} - T_{h2}) = C_c \cdot (T_{c2} - T_{c1})$ *(by heat balance)*

i.e. $$T_{c2} := T_{c1} + \frac{C_h \cdot (T_{h1} - T_{h2})}{C_c}$$

i.e. $T_{c2} = 68.664°C$ ***(exit temperature of cold fluid (water).)***

Case (ii): Counter-flow HX:

For Counter-flow HX, we have:

$$\varepsilon := \frac{1 - \exp(-NTU \cdot (1 - C))}{(1 - C \cdot \exp(-NTU \cdot (1 - C)))} \quad ...(12.56)$$

i.e. $\varepsilon = 0.543$ *(effectiveness of counter-flow HX)*

Compare this ε with $\varepsilon = 0.496$ obtained earlier.

Now, again, since hot fluid is the 'minimum' fluid, we have:

i.e. $T_{h2} := T_{h1} - \varepsilon \cdot (T_{h1} - T_{c1})$

i.e. $T_{h2} = 95.779°C$ ***(exit temperature of hot fluid (oil).)***

And, T_{c2} is obtained from:

$$T_{c2} := T_{c1} + \frac{C_h \cdot (T_{h1} - T_{h2})}{C_c}$$

i.e. $T_{c2} = 72.807°C$ ***...exit temperature of cold fluid (water).***

Note that as a result of increasing the cold fluid (water) flow rate, the new exit temperature of both the hot and cold fluids are lower, for both the parallel and counter-flow cases.

Example 12.10. A steam condenser, condensing at 70°C has to have a capacity of 100 kW. Water at 20°C is used and the outlet water temperature is limited to 45°C. If the overall heat transfer coefficient is 3100 W/m^{2}K, determine the area required.

(b) If the inlet water temperature is increased to 30°C, determine the increased flow rate of water to maintain the same outlet temperature. (M.U. 1998)

Solution. This problem can be solved by LMTD method, too. But, in part (b), since the heat exchanger is operated at an off-design condition, we shall adopt the ε–NTU method.

Data:

$Q := 100 \times 10^3$ W $\quad T_h := 70°C \quad U := 3100$ W/(m²K) $\quad T_{c1} := 20°C \quad T_{c2} := 45°C$

$C_{ph} := 1900$ J/(kgK) $\quad C_{pc} := 4180$ J/(kgK)

Therefore, effectiveness:

Since steam is condensing, it is the 'maximum' fluid. So, we can write:

$$\varepsilon := \frac{T_{c2} - T_{c1}}{T_h - T_{c1}}$$

i.e. $\varepsilon = 0.5$ *(effectiveness)*

Now, for a condenser, we have, from Table 12.6:

$\text{NTU} := -\ln(1 - \varepsilon)$ *(for a condenser)*

i.e. $\text{NTU} = 0.693$

and,
$$C_{\min} := \frac{Q}{\varepsilon \cdot (T_h - T_{c1})}$$

i.e. $C_{\min} = 4 \times 10^3$ W/K

But,
$$\text{NTU} = \frac{U \cdot A}{C_{\min}}$$ *(by definition of NTU)*

Therefore,
$$A := \frac{\text{NTU} \cdot C_{\min}}{U}$$

i.e. $A = 0.894$ **m²** ***(area of heat transfer.)***

Case (b): If T_{c1} is increased to 30°C, and T_{c2} maintained at 45°C, what is the increased flow rate?

$T_{c1} := 30°C$ *(new inlet temperature of water)*

Then,
$$\varepsilon := \frac{T_{c2} - T_{c1}}{T_h - T_{c1}}$$

i.e. $\varepsilon = 0.375$ *(new effectiveness)*

Therefore, new NTU:

$\text{NTU} := -\ln(1 - \varepsilon)$ *(for a condenser)*

i.e. $\text{NTU} = 0.47$ *(new value of NTU for case (b))*

Therefore,
$$C_{\min 2} := \frac{U \cdot A}{\text{NTU}}$$

i.e. $C_{\min 2} = 5.899 \times 10^3$ W/K *(new C_{min} for case (b))*

Compare this value with $C_{\min} = 4000$ obtained earlier.

Therefore, increased flow rate:

We have: $C_{pc} := 4180$ J/kgK *(specific heat for water)*

Therefore,
$$m_1 := \frac{C_{\min}}{C_{pc}}$$

i.e. $m_1 = 0.957$ kg/s *(earlier flow rate)*

and,
$$m_2 := \frac{C_{\min 2}}{C_{pc}}$$

$m_2 = 1.411$ **kg/s** ***(new flow rate)***

Also,
$$F := \frac{C_{\min 2}}{C_{\min}}$$

Or, $F = 1.475$ **(increase of 47.5%.)**

Example 12.11. Hot oil at a temperature of 180°C enters a shell and tube HX and is cooled by water entering at 25°C. There is one shell pass and 6 tube passes in the HX and the overall heat transfer coefficient is 350 W/(m^{2}K). Tube is thin-walled, 15 mm ID and length per pass is 5 m. Water flow rate is 0.3 kg/s and oil flow rate is 0.4 kg/s. Determine the outlet temperatures of oil and water and also the heat transfer rate in the HX. Given: specific heat of oil = 1900 J/(kgK) and specific heat of water = 4184 J/(kgK)

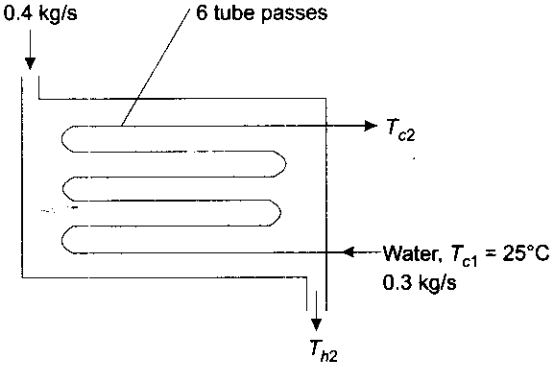


FIGURE Example 12.11 Shell and tube heat exchanger with one shell pass and 6 tube passes

Solution. Since the exit temperatures of both the fluids are not known, we shall use ε – NTU method.

Data:

$m_h := 0.4$ kg/s $m_c := 0.3$ kg/s $T_{h1} := 180°C$

$U := 350$ W/(m^{2}K) $T_{c1} := 25°C$ $N := 6$

$D := 0.015$ m $L := 5$ m $C_{ph} := 1900$ J/(kgK)

$C_{pc} := 4184$ J/(kgK)

Capacity rates:

$C_h := m_h \cdot C_{ph}$

i.e. $C_h = 760$ W/K

and, $C_c := m_c \cdot C_{pc}$

i.e. $C_c := 1.255 \times 10^3$ W/K

Therefore, oil is the 'minimum' fluid.

i.e. $C_{min} := C_h$ W/K *(minimum capacity rate)*

and, $C_{max} := C_c$ W/K *(maximum capacity rate)*

Therefore, capacity ratio:

$$C := \frac{C_{min}}{C_{max}}$$

i.e. $C := 0.605$ *(capacity ratio)*

Number of Transfer Units:

$$A := N \cdot \pi \cdot D \cdot L$$

i.e. $A = 1.414$ m^2 *(area of heat transfer)*

and, $$\text{NTU} := \frac{U \cdot A}{C_{min}}$$

i.e. NTU = 0.651 *(Number of Transfer Units)*

Effectiveness:

This is a shell and tube HX with one shell pass and 6 tube passes. So, its effectiveness can be determined for C = 0.605 and NTU = 0.651, from Fig. 12.14.

Since it is difficult to read from the graph accurately, let us calculate ε from analytical relation given in Table 12.5:

$N :=$ NTU *(notation in following equation)*

$$\varepsilon := 2 \cdot \left[1 + C + (1 + C^2)^{\frac{1}{2}} \cdot \frac{1 + \exp\left[-N \cdot (1 + C^2)^{\frac{1}{2}}\right]}{1 - \exp\left[-N \cdot (1 + C^2)^{\frac{1}{2}}\right]} \right]^{-1}$$ *(one shell pass, 2, 4 , 6 tube passes)*

i.e. $\varepsilon = 0.415$ *...effectiveness*

Actual heat transfer, Q:

We have: $Q = \varepsilon \cdot Q_{max}$

where, $Q_{max} := C_{min} \cdot (T_{h1} - T_{c1})$ W *(maximum possible heat transfer in the HX)*

i.e. $Q_{max} = 1.178 \times 10^5$ W

and, $Q := \varepsilon \cdot Q_{max}$

i.e. $Q = 4.884 \times 10^4$ **W** ***(actual heat transfer in the HX.)***

Outlet temperatures of hot and cold fluids:

We have: $Q = C_h \cdot (T_{h1} - T_{h2}) = C_c \cdot (T_{c2} - T_{c1})$

Therefore,

$$T_{h2} := T_{h1} - \frac{Q}{C_h}$$

i.e. $T_{h2} := 115.742°C$ *(**outlet temperature of hot fluid (i.e. oil)**)*

and,

$$T_{c2} := T_{c1} + \frac{Q}{C_c}$$

i.e. $T_{c2} = 63.907°C$ *(**outlet temperature of cold fluid (i.e. water)**.)*

Example 12.12. A feed water heater heats water entering at a temperature of 25°C, at a rate of 3 kg/s. Heating is due to steam condensing at 117°C. When the feed water heater was new (i.e. 'clean' condition), the exit temperature of water was 85°C. After prolonged operation, for the same flow rates and inlet conditions, it was observed that the outlet temperature was 75°C. Determine the value of fouling factor. Given: area of heat exchange = 5.5 m^2.

Solution. Fouling resistance, R_f is calculated from the relation:

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \text{ m}^2\text{K/W} \qquad \textit{(fouling factor)}$$

Also, since the steam is condensing, it is the 'maximum' fluid, and the water is the 'minimum fluid.

Data:

$m_c := 3$ kg/s $\quad T_h := 117°C \quad U := 350$ W/(m^{2}K) $\quad T_{c1} := 25°C$

$T_{c2} := 85°C$ *(exit temperature of cold fluid (water) for 'clean' HX)* $\quad C_{pc} := 4180$ J/(kgK) $\quad A := 5.5$ m^2

Capacity rates:

$C_c := m_c \cdot C_{pc}$ *(Capacity rate of cold fluid (water))*

i.e. $C_c = 1.25 \times 10^4$ W/K *(capacity rate of cold fluid (water))*

Since this is a condenser, water is the 'minimum' fluid, and capacity rate of condensing steam is ∞.

i.e. $C_{min} := C_c$

Therefore, capacity ratio:

$$C = \frac{C_{min}}{C_{max}}$$

i.e. $C = 0$ *(for a condenser)*

Effectiveness:

Remembering that water is the minimum fluid, effectiveness is given by:

$$\varepsilon := \frac{T_{c2} - T_{c1}}{T_h - T_{c1}}$$

i.e. $\varepsilon = 0.652$ *(effectiveness of the condenser)*

and, from Table 12.6, NTU of the condenser is given by:

$$\text{NTU} := -\ln(1 - \varepsilon)$$

i.e. $\text{NTU} = 1.056$

But, by definition of NTU:

$$\text{NTU} = \frac{U \cdot A}{C_{min}}$$

Therefore,

$$U_{clean} := \frac{\text{NTU} \cdot C_{min}}{A}$$

i.e. $U_{clean} = 2.408 \times 10^3$ W/(m^{2}K) *(overall heat transfer coefficient for 'clean' HX.)*

After prolonged operation:

$T_{c2} := 75°C$ *(exit temperature of water for 'dirty' HX.)*

Therefore, effectiveness of dirty HX:

$$\varepsilon := \frac{T_{c2} - T_{c1}}{T_h - T_{c1}}$$

i.e. $\varepsilon = 0.543$ *(effectiveness of 'dirty' HX.)*

Therefore, NTU of condenser:

$$\text{NTU} := -\ln(1-\varepsilon)$$

i.e. $\text{NTU} = 0.784$ *(for 'dirty' HX)*

Then, $$U_{\text{dirty}} := \frac{\text{NTU}\cdot C_{\min}}{A}$$

$U_{\text{dirty}} = 1.788 \times 10^3\ \text{W/(m}^2\text{K)}$ *(overall heat transfer coefficient for 'dirty' HX)*

Therefore, Fouling factor:

$$R_f := \frac{1}{U_{\text{dirty}}} - \frac{1}{U_{\text{clean}}}\ \text{m}^2\text{K/W}$$ *(fouling factor)*

i.e. $R_f = 0.000144$ **m²K/W** ***(fouling factor.)***

Example 12.13. Oil at 100°C (C_p = 3.6 kJ/kgK) flows at a rate of 30,000 kg/h and enters into a parallel-flow HX. Cooling water (C_p = 4.2 kJ/kg.K) enters the HX at 10°C at the rate of 50,000 kg/h. The heat transfer area is 10 m² and U = 1000 W/(m²K). Calculate the following: (i) outlet temperature of oil and water (ii) maximum possible outlet temperature of water.

Solution. Exit temperature of both the fluids are not known; therefore, NTU method is to be used:

Data:

$m_h := \frac{30000}{3600}$ kg/s i.e. $m_h = 8.333$ $\quad m_c := \frac{50000}{3600}$ kg/s i.e. $m_c = 13.889$ $\quad T_{h1} := 100°\text{C}$ $\quad U := 1000\ \text{W/(m}^2\text{K)}$

$T_{c1} := 10°\text{C}$ $\quad C_{ph} := 3600\ \text{J/(KgK)}$ $\quad C_{pc} := 4200\ \text{J/(kgK)}$ $\quad A := 10\ \text{m}^2$

Capacity rates:

$$C_h := m_h \cdot C_{ph}$$

i.e. $C_h = 3 \times 10^4\ \text{W/K}$

and, $C_c := m_c \cdot C_{pc}$

i.e. $C_c = 5.833 \times 10^4\ \text{W/K}$

Therefore, oil is the 'minimum' fluid.

i.e. $C_{\min} := C_h\ \text{W/K}$ *(minimum capacity rate)*

and, $C_{\max} := C_c\ \text{W/K}$ *(maximum capacity rate)*

Therefore, Capacity ratio:

$$C := \frac{C_{\min}}{C_{\max}}$$

i.e. $C = 0.514$ *(capacity ratio)*

and, $$\text{NTU} := \frac{U\cdot A}{C_{\min}}$$

i.e. $\text{NTU} = 0.333$ *(Number of Transfer Units)*

Effectiveness:

For parallel flow HX, we have:

$$\varepsilon := \frac{1-\exp(-\text{NTU}\cdot(1+C))}{1+C} \quad \text{...(12.51)}$$

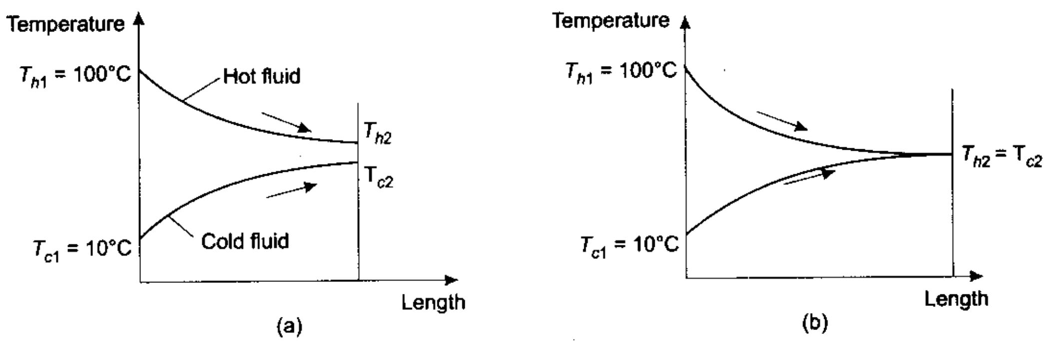


FIGURE Example 12.13 Parallel-flow heat exchanger i.e. $\varepsilon = 0.262$ *(effectiveness of parallel flow HX with NTU = 0.333 and C = 0.514.)*

Note: We can use the graph of Fig. 12.10 or Table 12.7, but using the analytical relation is more accurate.

Outlet temperature of hot and cold fluids:

Since hot fluid is the minimum fluid, $\varepsilon = \dfrac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$

Therefore,

$$T_{h2} := T_{h1} - \varepsilon \cdot (T_{h1} - T_{c1})$$

i.e. $T_{h2} = 76.443°\text{C}$ ***(outlet temperature of hot fluid (i.e. oil).)***

And, from heat balance:

$$C_h \cdot (T_{h1} - T_{h2}) = C_c \cdot (T_{c2} - T_{c1})$$

Or, $$T_{c2} := T_{c1} + \frac{C_h \cdot (T_{h1} - T_{h2})}{C_c}$$

i.e. $T_{c2} = 22.115°\text{C}$ ***(outlet temperature of cold fluid (i.e. water).)***

(b) Maximum possible outlet temperature of water:

For a very long parallel-flow HX, the outlet temperatures of hot and cold fluids would be the same:

i.e. $T_{h2} = T_{c2}$

Therefore, writing the heat balance:

$$C_h \cdot (T_{h1} - T_{h2}) = C_c \cdot (T_{c2} - T_{c1})$$

i.e. $$C_h \cdot (T_{h1} - T_{c2}) = C_c \cdot (T_{c2} - T_{c1})$$

i.e. $$T_{c2} \cdot (C_c + C_h) = C_c \cdot T_{c1} + C_h \cdot T_{h1}$$

i.e. $$T_{c2} := \frac{C_c \cdot T_{c1} + C_h \cdot T_{h1}}{C_c + C_h}$$

i.e. $T_{c2} = 40.566°\text{C}$ ***(maximum possible outlet temperature of water.)***

12.7 The Operating-line/Equilibrium-line Method

NTU–ε method can be represented graphically in another way.

Refer to Fig. 12.16. Here, the x-axis represents the cold fluid temperature and the y-axis, the hot fluid temperature. Now, if we plot the entrance and exit temperatures of a heat exchanger on these axes, we see that the operating range of the HX is represented by a single line; this line is called 'the operating line'. On the same graph, a line drawn at 45 deg. is called 'the equilibrium line'. For equilibrium line, $T_h = T_c$. Thermodynamically, it is impossible for the operating line of a heat exchanger to drop below the equilibrium line, since, if it does, it would mean a violation of the second law. Slope of the operating line for the counter-flow HX is: $(C_c/C_h) = (T_{h1} - T_{h2})/(T_{c2} - T_{c1})$. And, the slope of the operating line for the parallel-flow HX is: $-(C_c/C_h)$, i.e. negative slope. For a condenser, operating line is horizontal with T_h = constant and $(C_c/C_h) = 0$, and for an evaporator, the operating line is a vertical line with T_c = constant, and $(C_c/C_h) = \infty$.

Advantage of this method of representation is that the effectiveness of the heat exchanger can now be shown geometrically as a ratio of two lengths. For example, for the counter-flow HX shown in Fig. 12.16 (a), we have: $C_c > C_h$ and the effectiveness is equal to δ/Δ.

For constant specific heats of fluids, the operating line is a straight line. Variation in specific heats of fluids is also shown easily in these graphs: As shown in Fig. 12.16 (d), if the operating line curves upwards, i.e. the slope increases as the temperature increases, it means that C_p of cold fluid increases with temperature (or, the C_p of hot fluid decreases with temperature). Similarly, if the operating line curves downwards, it means that C_p of cold fluid decreases with temperature (or, the C_p of hot fluid increases with temperature).

Effectiveness of a parallel-flow heat exchanger:

Operating-line/Equilibrium line method can be used to determine the effectiveness of a heat exchanger. Let us illustrate this briefly with reference to a parallel-flow HX:

Refer to Fig. 12.17. Line 1–2 is the operating line for the parallel-flow HX. We see from the figure that $C_c < C_h$, since the slope of the operating line = $-C_c/C_h$.

And, Capacity ratio, $C = C_c/C_h$.

From the Fig 12.17:

$$T_{h1} - T_{c1} = \Delta$$
$$T_{c2} - T_{c1} = a_1$$

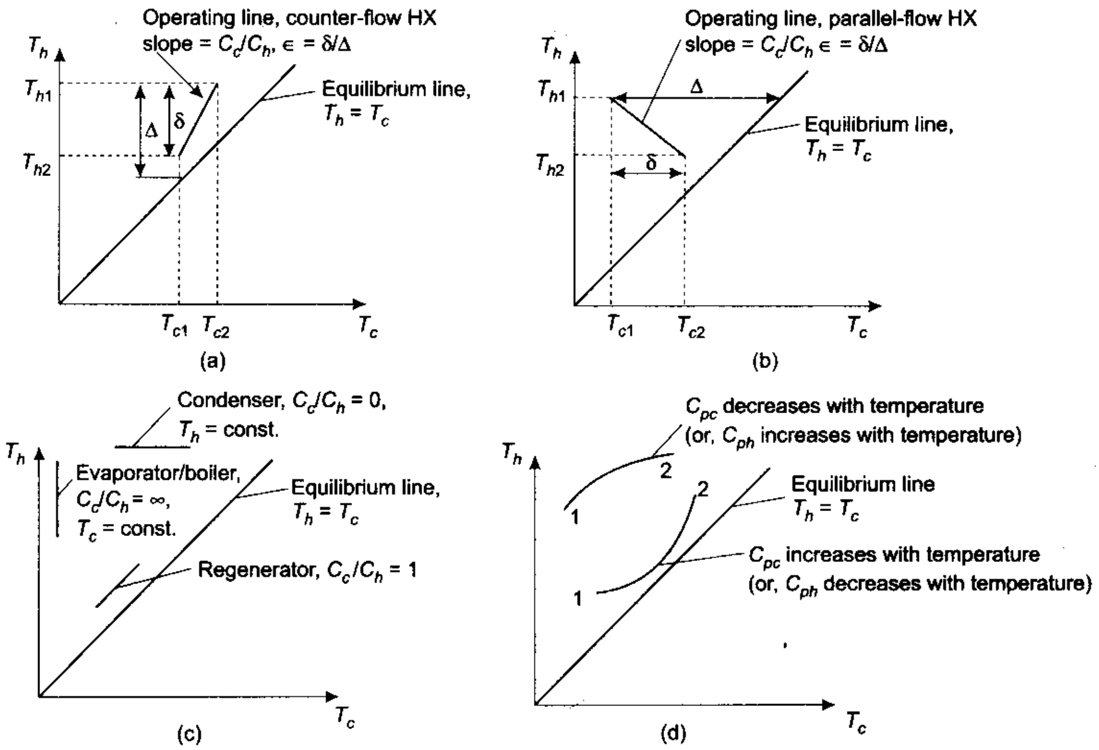


FIGURE 12.16 Operating line and equilibrium lines for heat exchangers

Therefore, $\varepsilon = \dfrac{a_1}{\Delta}$

For parallel-flow HX, we have:

$$\ln\left(\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}}\right) = -U\cdot A\cdot\left(\frac{1}{m_h\cdot C_{ph}}+\frac{1}{m_c\cdot C_{pc}}\right) \quad \text{...(12.21)}$$

i.e. $$\ln\left(\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}}\right) = -U\cdot A\cdot\left(\frac{1}{C_h}+\frac{1}{C_c}\right) =$$

$$\frac{-U\cdot A}{C_c}\cdot(1+C) \quad \text{...(A)}$$

Now, from the Fig. 12.17 we have:

$T_{h2} - T_{c2} = b_1$

$T_{h1} - T_{c1} = \Delta$

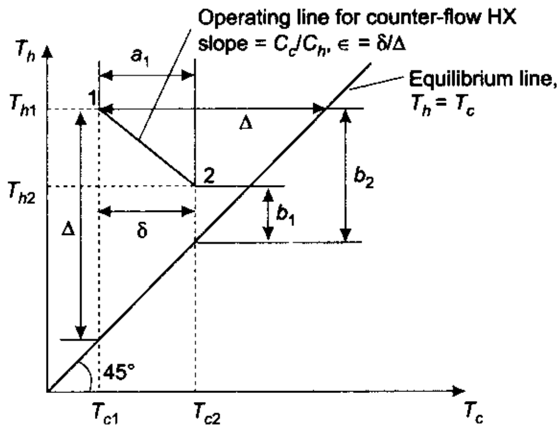


FIGURE 12.17 Parallel-flow heat exchanger

We also see from the Fig. 12.17:

$$b_1 = b_2 - (T_{h1} - T_{h2})$$

But, $b_2 = \Delta - a_1$ *(from the Fig. 12.17, since equilibrium line is at 45 deg. to horizontal.)*

Therefore, $b_1 = (\Delta - a_1) - (T_{h1} - T_{h2})$

i.e. $$b_1 = (\Delta - a_1) - \frac{C_c}{C_h}\cdot a_1$$

Therefore, $$\frac{b_1}{\Delta} = 1 - \frac{a_1}{\Delta} - \frac{C_c}{C_h}\cdot\frac{a_1}{\Delta}$$

i.e. $$\frac{b_1}{\Delta} = 1 - \varepsilon - C \cdot \varepsilon$$

i.e. $$\frac{b_1}{\Delta} = 1 - \varepsilon \cdot (1 + C) \qquad \text{...(B)}$$

Substituting in Eq. A:

$$\frac{b_1}{\Delta} = \exp(-\text{NTU} \cdot (1 + C))$$

Using Eq. B:

$$1 - \varepsilon \cdot (1 + C) = \exp(-\text{NTU} \cdot (1 + C))$$

i.e. $$\varepsilon = \frac{1 - \exp(-\text{NTU} \cdot (1 + C))}{1 + C} \qquad \text{...(C)}$$

Eq. C is the desired equation for the effectiveness of the parallel-flow HX.

This is the same as the equation derived earlier for parallel-flow HX, i.e. Eq. 12.51. While deriving Eq. C, it was assumed that the cold fluid was the 'minimum' fluid; if we assume that the hot fluid is the minimum fluid, then also, the same result would be obtained.

12.8 Compact Heat Exchangers

Heat exchangers with an area density greater than about 700 m^2/m^3 are classified as 'compact heat exchangers'. Generally, they are used for gases.

Compact heat exchangers are, typically, of three types:

(i) array of finned circular tubes
(ii) array of plate-fin matrix, and
(iii) array of finned flat-tube matrix.

Heat transfer and pressure drops for these compact heat exchangers are determined experimentally and are supplied by manufacturers as their proprietary data.

As an example, a plate-fin type of heat exchanger matrix, manufactured by Marston–Excelsior Ltd., is shown in Fig. 12.18. As shown in the Fig.12.18, a single element consists of two plates in between which is sandwiched a corrugated sheet. The two edges are sealed. Dip brazing technique is used to build a complete heat exchanger block from individual elements. Multi-flow configurations are possible, and the generally used corrugations types are: plain (P), plain-perforated (R), serrated (S) and herringbone (H).

Table 12.9 gives the geometrical data for some typical corrugations.

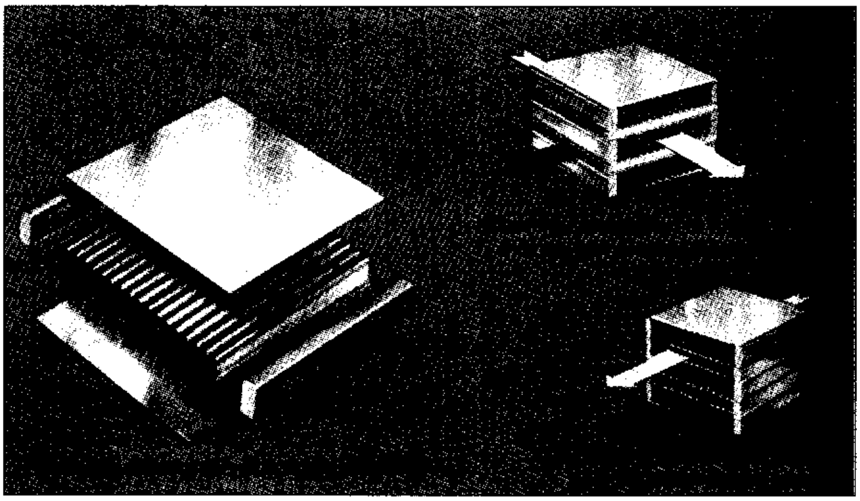


FIGURE 12.18 Plate-fin heat exchangers for cryogenic service (Marston–Excelsior Ltd.)

TABLE 12.9 Geometrical data for typical corrugations (Marston Excelsior Ltd.)

Type	Height (mm)	Thickness (mm)	Pitch (fins/m)	a (m^2/m)	A_1(m^2/m)	A_2(m^2/m)	D_h(mm)
P, R, S, H	3.8	0.20	470	0.00326	1.81	3.41	2.5
P, R	5.1	0.20	550	0.004328	1.776	5.376	2.42
P, R	5.1	0.20	1020	0.00387	1.584	9.984	1.34
P, R, S, H	6.35	0.30	470	0.005175	1.712	5.712	2.79
P, R, H	8.9	0.46	590	0.00615	1.46	9.96	2.16
P, R	8.9	0.61	240	0.00707	1.712	3.912	5.04

In the above table,

a = free flow area per metre width of corrugation

A_1 = (primary surface area per metre width) x (metre length of corrugation)

A_2 = (secondary surface area per metre width) x (metre length of corrugation)

D_h = hydraulic mean diameter

i.e.

$$D_h = \frac{4 \cdot a}{\text{wetted perimeter}}$$

i.e.

$$D_h = \frac{4 \cdot a}{(A_1 + A_2)} \qquad \text{(for areas specified above.)}$$

Kays and London have studied a large number of compact heat exchanger matrices and presented their experimental results in the form of generalised graphs. Heat transfer data is plotted as $St.Pr^{2/3}$ against Re, where, St = Stanton number = $h/(G.C_p)$, Pr = Prandtl number = $\mu.C_p/k$, and $Re = G.D_h/\mu$, G = mass velocity (= mass flow rate/Area of cross section), kg/(sm^2.)

In the same graphs, friction factor, f, is also plotted against Re.

As an example, heat transfer and friction factor characteristics for a particular tube-fin matrix are shown in Fig.12.19.

Pressure drop in plate-fin heat exchangers:

Total pressure drop for the fluid flowing across the heat exchanger is given by:

$$\Delta P = \frac{G^2}{2 \cdot \rho_i} \cdot \left[(K_c + 1 - \sigma^2) + 2 \cdot \left(\frac{\rho_i}{\rho_o} - 1 \right) + f \cdot \frac{A}{A_{\min}} \cdot \frac{\rho_i}{\rho_m} - (1 - K_e - \sigma^2) \cdot \frac{\rho_i}{\rho_o} \right] \qquad \text{...(12.59)}$$

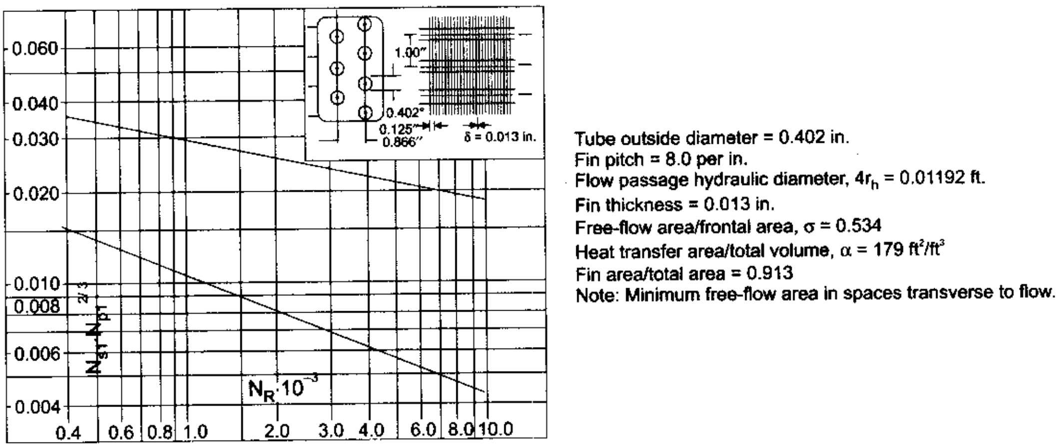


FIGURE 12.19 Heat transfer and friction factor for plate-finned circular tube matrix (Trane Company)

Above equations are simplified as:

$$\partial t/\partial \tau = \{(h.A)/(C_{ps}.M_s)\}.(t_g - t) \quad ...(12.63)$$

$$\partial t_g/\partial x + (\rho.V/M).\partial t_g/\partial \tau = \{(h.A)/(C_{pg}.M)\}.(t - t_g) \quad ...(12.64)$$

In most of the practical situations, the term $(\rho.V/M)$ is very small and is neglected. Then, making following substitutions

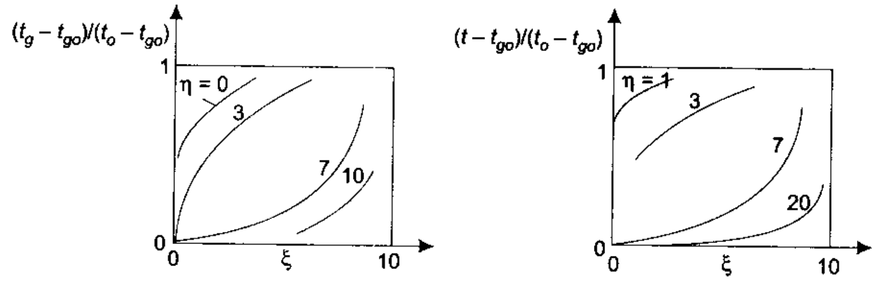
$$\xi = \frac{h \cdot A \cdot x}{C_{pg} \cdot M} \quad \text{and} \quad \eta = \frac{h \cdot A}{C_{ps} \cdot M_s} \cdot \tau$$

the resulting equations are solved with the following boundary conditions:

$t = t_o$ at $\eta = 0$ *(initial solid temperature for all ξ)*

and, $t_g = t_{go}$ at $\xi = 0$ *(inlet gas temperature for all η)*

The results are presented usually in graphical form and the nature of graphs is shown in Fig. 12.23 (a) and (b).



(a) To find gas temperature

(b) To find solid temperature

FIGURE 12.23 Gas and solid temperature charts for a regenerator

In these graphs, t_{go} is the initial temperature of the gas, and t_o is the initial temperature of the solid.

Fig. 12.23 (a) presents the dimensionless gas temperature at any location as a function of ξ and η and Fig. 12.23 (b) shows the dimensionless solid temperature as a function of ξ and η.

Effectiveness–NTU relations for regenerator:

Effectiveness of a regenerator is presented as a function of three dimensionless parameters, as follows:

$$\varepsilon = f\left(\text{NTU}_{\text{mod}}, \frac{C_{\min}}{C_{\max}}, \frac{C_r}{C_{\min}}\right) \quad ...(12.65)$$

where, NTU_{mod} = modified NTU, given by:

$$\text{NTU}_{\text{mod}} = \frac{1}{C_{\min}} \cdot \left[\frac{1}{\left(\frac{1}{h \cdot A}\right)_c + \left(\frac{1}{h \cdot A}\right)_h}\right] \quad ...(12.66)$$

and, matrix capacity rate is equal to matrix mass rate times the specific heat of the solid.

For the rotary type of regenerator,

$$C_r = \left(\frac{Rev}{s}\right) \cdot (\text{matrix mass}) \cdot C_{ps} \text{ W/K} \quad \text{...for rotary type regenerator...}(12.67).$$

For the valved type of regenerator, total mass of both the identical matrices is used, multiplied by valve cycles/s, where period is the interval between 'valve–on–to–off–to–on'.

Kays and London have presented ε-NTU_{mod} graphs for different $C_r/C_{\min}$ ratios (ranging from 1 to infinity), for given $C_{\min}/C_{\max}$ ratios (ranging from 0.5 to 1). Table 12.10 is a sample table showing ε values for $C_{\min}/C_{\max}$ = 1. Fig. 12.24 presents this table in graphical form.